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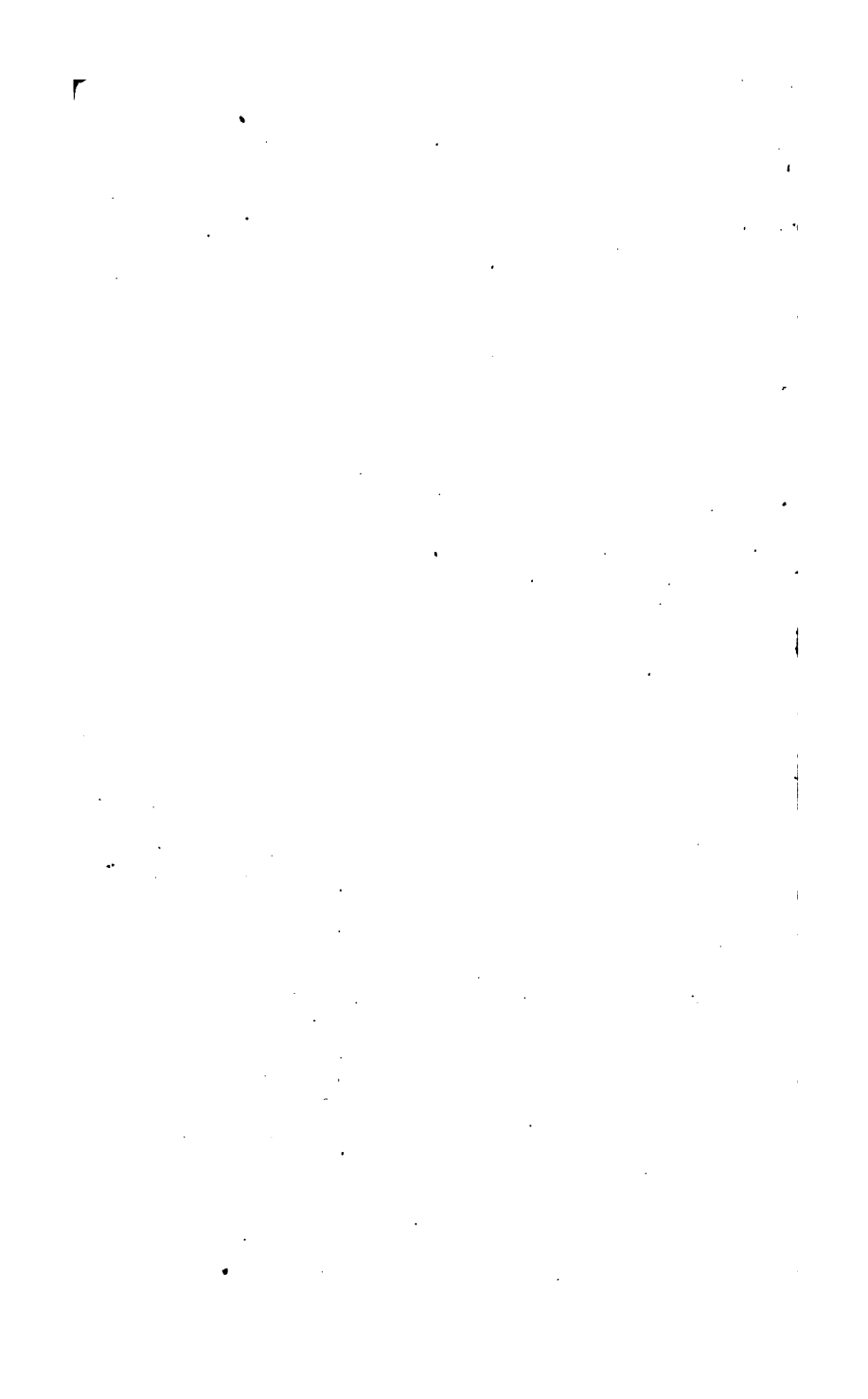
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K E Y
TO
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BY THE REV. E. C. TYSON, M.A.
FELLOW OF CATHARINE HALL, CAMBRIDGE, AND LATE HEAD MASTER IN THE ROYAL
MATHEMATICAL SCHOOL, CHRIST'S HOSPITAL.

A NEW EDITION, ENTIRELY RE-EDITED.

By Dr. RUTHERFORD, F.R.A.S.
ROYAL MILITARY ACADEMY.



LONDON:
WILLIAM TEGG, 85, QUEEN STREET, CHEAPSIDE.
1860.

181. c. 6.



LONDON: PRINTED BY WILLIAM CLOWES AND SONS, STAMFORD STREET.

PREFACE.

THIS Key to Bonnycastle's Guide to Arithmetic contains the solutions to all the unanswered questions embodied in that excellent work. In preparing this edition of the Key, every solution has been examined with care; some faulty solutions have been corrected, others have been modified, and answers have been supplied to all the exercises which were left unanswered in the previous edition, as presenting difficulties unsurmountable without the introduction of algebraic symbols and investigations which were deemed foreign to a work on arithmetic. Some of the exercises in the Rule of Position, and in the "Arithmetical Recreations" especially, were considered beyond the power of arithmetic to solve; but to several of these, purely arithmetical solutions have been given, which, it is hoped, will be interesting to the student.

The modern method of extracting the cube root of any number has been introduced in this edition, thereby rendering the operation nearly as simple as that for the extraction of the square root.

The Work, together with the Key, forms an excellent treatise on the science of Arithmetic, one of the most useful branches of education; and as the Key has been enriched with several improved processes of calculation, it is hoped that, jointly, they will supply an amount of information seldom met with in any of the ordinary treatises on Arithmetic.

WILLIAM RUTHERFORD.

*Royal Military Academy,
Woolwich.*

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KEY

TO

BONNYCASTLE'S ARITHMETIC.

NUMERATION.

Answers to the Examples in pages 2 and 3.

52—185—204—717—860—303—1033—157505—
1300000—1200675—2000097—808008008.

Fifty-eight—One hundred and one—One thousand, seven hundred and eighty-six—One thousand, eight hundred and twenty-eight—One hundred and one thousand, and eighteen—Five hundred and five thousand, six hundred and eighty-one—Nine thousand, and ninety-one—Ten thousand, eight hundred and three—Forty-two thousand, six hundred and seventy-eight—One hundred and thirty thousand, three hundred and six—One hundred and twenty-three thousand, four hundred and fifty-six—One million, two hundred and thirty-four thousand, five hundred and sixty-seven—Thirty-eight thousand, seven hundred and twenty-six—Five hundred thousand, and two—Four hundred and one thousand, and nineteen—Seven hundred and sixty-eight thousand, five hundred and twenty—Thirteen thousand, five hundred and seventy-nine—Nine millions, nine hundred and eight thousand, and sixty.

SIMPLE ADDITION, p. 5.

(4)	(5)	(6)	(7)
8653	97684	3684	37006
2194	30768	4863	429009
7421	5015	365	2000036
5063	307	29	<u>2466051</u>
2196	36	56874	
1245	15	609	
<u>26772</u>	<u>133825</u>	<u>66424</u>	

(8)	(9)	(10)
January . 31 days.	April . . 29 days.	£ 3129
February . 29 „	May . . 31 „	5009
March . . 31 „	June . . 30 „	4009
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July . . 31 „	October . 31 „	<u>£ 24174</u>
August . 31 „	November 28 „	
September 30 „	1 Year = 365 „	
<u>274 days.</u>	<u>606 days.</u>	

SIMPLE SUBTRACTION, p. 6.

(4)	(5)
2683947	1060901
685427	999999
<u>1998520</u>	<u>60902</u>

(6)	Years.
The Indian method of notation was first known	1859
in England about the year 1150; how many	1150
years have elapsed from that period to the	<u>Ans. 709</u>
present year 1859?	

(7)	Years.
Sir Isaac Newton was born in the year 1642,	1727
and died in 1727; how many years did he	1642
live?	<u>Ans. 85</u>

(8)

The art of printing was invented in . . . 1440
 and the mariner's compass in . . . 1302
 Interval = 138 years.

Columbus discovered America in . . . 1492
 The mariner's compass invented in . . . 1302
 Interval = 190 years.

Columbus discovered America in . . . 1492
 The art of printing discovered in . . . 1440
 Interval = 52 years.

(9)

Distance between London and Edinburgh . 393 miles.
 " " London and Newcastle . 272 "
 Distance between Edinburgh and Newcastle 121 miles.

(10)

Years.

The Peloponnesian war began about 431 years . 2290
 before the birth of our Saviour, and 2290 . 431
 years from the present year; required the *Ans.* 1859
 number of years elapsed since the birth of
 Christ.

(11)

Years.

The dedication of the temple at Jerusalem took
 place before the Christian era 1004
 Jerusalem was conquered before ditto 587
 Interval = 417

(12)

The first recorded lunar eclipse took place 2580 years.
 before the present year 1859 "
 Interval between eclipse and birth of Christ = 721 years.

(13)

Homer was born 2766 years ago.
 Homer lived 62 years.
 Homer died 2704 years ago.
 Birth of our Saviour 1859 "
 Interval between the death of Homer }
 and the birth of our Saviour . . } = 845 years.

SIMPLE MULTIPLICATION, p. 9.

(2) 4768923 7 <u>33382461</u>	(3) 796534289 11 <u>Prod. 8761877179</u>	(4) 54328432847 8 <u>Prod. 434627462776</u>
(5) 8643597 9 <u>77792373</u>	(6) 57864329 579 <u>520778961</u> 405050303 289321645 <u>Prod. 33503446491</u>	(7) 3274656461 12 <u>Prod. 39295877532</u>
(8) 7324687567 15 <u>36623437835</u> 7324687567 <u>109870313505</u>	(9) 94713761 18 <u>757710088</u> 94713761 <u>Prod. 1704847698</u>	(10) 273580961 23 <u>820742883</u> 547161922 <u>Prod. 6292362103</u>
(11) 27501976 271 <u>27501976</u> 192513832 55003952 <u>7453035496</u>	(12) 82164973 3027 <u>575154811</u> 164329946 246494919 <u>248713373271</u>	(13) 6247386495 27356 <u>37484318970</u> 31236932475 18742159485 43731705465 12494772990 <u>Prod. 170903504957220</u>
(14) 8496427 874359 <u>76467843</u> 42482135 25489281 33985708 59474989 67971416 <u>Prod. 7428927415293</u>	(15) 123456789 123456789 <u>1111111101</u> 987654312 864197523 740740734 617283945 493827156 870370367 246913578 123456789 <u>Prod. 15241578750190521</u>	

CONTRACTION—I.

$$\begin{array}{r}
 (3) \\
 461200 \\
 \underline{72000} \\
 9224 \\
 32284 \\
 \underline{\hspace{1.5cm}} \\
 33206400000
 \end{array}$$

$$\begin{array}{r}
 (4) \\
 815036000 \\
 \underline{70300} \\
 2445108 \\
 5705252 \\
 \underline{\hspace{1.5cm}} \\
 57297030800000
 \end{array}$$

CONTRACTION—II.

$$\begin{array}{r}
 (2) \\
 785432 \\
 \underline{6 \times 6 = 36} \\
 4712592 \\
 \underline{6} \\
 28275552
 \end{array}$$

$$\begin{array}{r}
 (4) \\
 364111 \\
 \underline{8 \times 7 = 56} \\
 2912888 \\
 \underline{7} \\
 20390216
 \end{array}$$

$$\begin{array}{r}
 (6) \\
 7128368 \\
 \underline{12 \times 8 = 96} \\
 85540416 \\
 \underline{8} \\
 684323328
 \end{array}$$

$$\begin{array}{r}
 (3) \\
 472849 \\
 \underline{5 \times 9 = 45} \\
 2364245 \\
 \underline{9} \\
 21278205
 \end{array}$$

$$\begin{array}{r}
 (5) \\
 4612319 \\
 \underline{9 \times 8 = 72} \\
 41510871 \\
 \underline{8} \\
 332086968
 \end{array}$$

$$\begin{array}{r}
 (7) \\
 5416879 \\
 \underline{12 \times 12 = 144} \\
 65002548 \\
 \underline{12} \\
 780030576
 \end{array}$$

SIMPLE DIVISION, p. 13.

$$\begin{array}{r}
 (4) \\
 3) 5474857647651 \\
 \underline{1824952549217} \text{ Quot.}
 \end{array}$$

$$\begin{array}{r}
 (5) \\
 4) 653783754732 \\
 \underline{163445938683} \text{ Quot.}
 \end{array}$$

$$\begin{array}{r}
 (6) \\
 6) 2345678964 \\
 \underline{390946494} \text{ Quot.}
 \end{array}$$

$$\begin{array}{r}
 (7) \\
 7) 12345678900 \\
 \underline{1763668414} \text{ Quot.}
 \end{array}$$

$$\begin{array}{r}
 (8) \\
 8) 9876543210 \\
 \underline{1234567901\frac{1}{8}}
 \end{array}$$

$$\begin{array}{r}
 (9) \\
 9) 1357975313 \\
 \underline{150886145\frac{1}{9}}
 \end{array}$$

$$\begin{array}{r}
 (10) \\
 12) 570196382 \\
 \underline{47516365\frac{1}{12}}
 \end{array}$$

$$17 \overline{) 3217684329765} \quad (11) \quad 189275548809 \frac{1}{7} \text{ Quotient.}$$

$$\begin{array}{r} 17 \\ \hline 151 \\ 186 \\ \hline 157 \\ 153 \\ \hline 46 \\ 84 \\ \hline 128 \\ 119 \\ \hline 94 \\ 85 \\ \hline 93 \\ 85 \\ \hline 82 \\ 68 \\ \hline 149 \\ 136 \\ \hline 137 \\ 136 \\ \hline 165 \\ 153 \\ \hline 12 \end{array}$$

$$27 \overline{) 321147368} \quad (12) \quad 11894346 \frac{1}{7} \text{ Quot.}$$

$$\begin{array}{r} 27 \\ \hline 51 \\ 27 \\ \hline 241 \\ 216 \\ \hline 254 \\ 243 \\ \hline 117 \\ 108 \\ \hline 93 \\ 81 \\ \hline 126 \\ 108 \\ \hline 188 \\ 162 \\ \hline 26 \end{array}$$

$$97 \overline{) 137896254} \quad (13) \quad 1421610 \frac{1}{7}. \quad 108 \overline{) 1406878} \quad (14) \quad 13021 \frac{1}{7}.$$

$$\begin{array}{r} 97 \\ \hline 408 \\ 388 \\ \hline 209 \\ 194 \\ \hline 156 \\ 97 \\ \hline 592 \\ 582 \\ \hline 105 \\ 97 \\ \hline 84 \end{array}$$

$$\begin{array}{r} 108 \\ \hline 326 \\ 324 \\ \hline 237 \\ 216 \\ \hline 213 \\ 108 \\ \hline 105 \end{array}$$

SIMPLE DIVISION.

7

(15)

345) 3405657254 (9871470 ~~111~~ Quotient.3105

(16)

3006

678) 5713070046 (8426357 Quot.

27605424

2465

289024152712

507

1787

3451356

1622

4310

13804068

2425

2420

24152034104

3864

3390

4746

4746

(17)

8405) 293839455936 (34960078 ~~1111~~ Quotient.25215

41689

(18)

33620

80694

57606) 4637064283 (80496 ~~11111~~75645460848

50495

285842

50430230424

65593

554188

58835518454

67586

357343

6724034563634611707

SIMPLE DIVISION.

(19)

210472) 352107193214 (1672940 Quotient.

$$\begin{array}{r}
 210472 \\
 \hline
 1416351 \\
 1262832 \\
 \hline
 1535199 \\
 1473304 \\
 \hline
 618953 \\
 420944 \\
 \hline
 1980092 \\
 1894248 \\
 \hline
 858441 \\
 841888 \\
 \hline
 165534 \text{ Remainder.}
 \end{array}$$

(20)

2708630425) 558001172606176724 (206008604 Quot.

$$\begin{array}{r}
 5417260850 \\
 \hline
 16275087606 \\
 16251782550 \\
 \hline
 23305056176 \\
 21669043400 \\
 \hline
 16360127767 \\
 16251782550 \\
 \hline
 10834521724 \\
 10834521700 \\
 \hline
 24 \text{ Remainder.}
 \end{array}$$

(21)

23017) 267000005 (11600 ¹¹⁶⁰⁰/₁₁₆₀₁₇ Quotient.

$$\begin{array}{r}
 23017 \\
 \hline
 36830 \\
 23017 \\
 \hline
 138130 \\
 138102 \\
 \hline
 2805
 \end{array}$$

$$\begin{array}{r}
 (22) \\
 30000 \\
 \underline{625} \quad \text{£.} \\
 18) 29375 (1631 \frac{1}{2} \\
 \underline{18} \\
 113 \\
 \underline{108} \\
 57 \\
 \underline{54} \\
 35 \\
 \underline{18} \\
 17
 \end{array}$$

$$\begin{array}{r}
 (23) \\
 5904 \\
 \underline{123} \\
 5781 = \text{greater No.} \\
 \underline{123} = \text{less No.} \\
 17343 \\
 \underline{69372} \\
 711063 = \text{Product.}
 \end{array}$$

$$\begin{array}{r}
 5781 \\
 \underline{123} \\
 5658 = \text{diff.}
 \end{array}$$

$$\begin{array}{r}
 123) 5781 (47 = \text{Quot.} \\
 \underline{492} \\
 861 \\
 \underline{861}
 \end{array}$$

CONTRACTIONS.

I. When ciphers are annexed to the divisor.

$$\begin{array}{r}
 (3) \\
 23,000) 7380,964 (320 \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \text{ Quotient.} \\
 \underline{69} \\
 48 \\
 \underline{46} \\
 20
 \end{array}$$

$$\begin{array}{r}
 (4) \\
 35,000) 29628754,963 (846535 \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \text{ Quotient.} \\
 \underline{280} \\
 162 \\
 \underline{140} \\
 228 \\
 \underline{210} \\
 187 \\
 \underline{175} \\
 125 \\
 \underline{105} \\
 204 \\
 \underline{175} \\
 29
 \end{array}$$

II. When the divisor is the product of two or more numbers that do not exceed 12.

$$\begin{array}{r}
 \begin{array}{l} (2) \\ 72 \left\{ \begin{array}{l} 6) 7014596 \\ 12) 1169099 \end{array} \right. \end{array} \\
 \hline
 97424 - 11 \\
 \hline
 6 \\
 66 \\
 \text{Add } 2 \\
 \text{Rem.} = 68
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{l} (3) \\ 132 \left\{ \begin{array}{l} 12) 5130652 \\ 11) 427554 \end{array} \right. \end{array} \\
 \hline
 38868 - 6 \\
 \hline
 12 \\
 72 \\
 \text{Add } 4 \\
 \text{Rem.} = 76
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{l} (4) \\ 240 \left\{ \begin{array}{l} 20) 83016572 \\ 12) 4150828 \end{array} \right. \end{array} \\
 \hline
 345902 - 4 \\
 \hline
 20 \\
 80 \\
 \text{Add } 12 \\
 \text{Rem.} = 92
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{l} (4) \\ 240 \left\{ \begin{array}{l} 10) 83016572 \\ 4) 8301657 \\ 6) 2075414 \end{array} \right. \end{array} \\
 \hline
 345902 - 2 \\
 \hline
 4 \\
 9 \\
 10 \\
 \text{Rem.} = 92
 \end{array}$$

III. To perform division more concisely than by the general rule.

$$\begin{array}{r}
 \begin{array}{l} (2) \\ 5317) 29137062 (5479\frac{1}{2} \\ 25520 \\ 42526 \\ 53072 \\ 5219 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{l} (3) \\ 7803) 62015735 (7947\frac{1}{3} \\ 73947 \\ 37203 \\ 59915 \\ 5294 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{l} (4) \\ 873469) 432756284563574 (495445498\frac{1}{4} \\ 8336868 \\ 4756474 \\ 3891295 \\ 3974196 \\ 4803203 \\ 4358585 \\ 8647097 \\ 7858764 \\ 871012 \end{array}
 \end{array}$$

COMPOUND ADDITION, pp. 17 & 18.

	£.	s.	d.		£.	s.	d.		£.	s.	d.
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	5	17	7½		109	18	10		325	17	2
	0	18	8		730	10	1½		840	12	9½
	0	17	0		185	14	2		426	17	8½
Sum	2009	9	10		2162	8	6½		2998	2	4½
	733	17	6		1933	14	0½		2319	8	10
Proof	2009	9	10		2162	8	6½		2998	2	4½

£.	s.	d.	£.	s.	d.	£.	s.	d.
678	5	10	368	10	3	567	8	9
87	10	9½	257	10	5	259	16	8½
123	8	8	88	11	4½	287	16	7½
47	16	9	33	10	0	87	15	4
307	2	0	12	13	5	25	16	8
187	16	10½	8	8	8½	24	10	2
1432	0	11	769	4	2	1253	4	3½

£.	s.	d.		£.	s.	d.
1728	10	8½	For Bread . . .	9	6	3½
457	10	6	Cheese . . .	4	3	0
328	19	9½	Tea . . .	10	9	5
478	12	2½	Butter . . .	3	0	2½
238	14	10	Sugar . . .	125	0	2½
50	10	6½	Other articles . .	26	13	6½
3822	18	7	Ans.	178	12	8½

£.	s.	d.	£.	s.	d.	£.	s.	d.
16	12	2	150	6	5½	458	10	11½
156	9	9½	29	13	4½	1087	2	3½
203	1	1½	15	19	11½	942	16	7½
52	10	0	118	2	1½	1234	5	6
101	0	0	54	10	3	114	13	8
77	7	7½	72	8	7½	29	19	11½
607	0	8½	441	0	9	3867	8	11½

TROY WEIGHT, p. 20.

lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.
171	6	13	14	49	8	7	10	71	11	19	
391	11	9	12	56	3	13	23	64	8	14	
230	6	6	13	99	11	19	1	77	0	0	
94	7	3	18	9	9	9	19	14	3	11	
42	10	15	20	9	10	3	2	64	2	9	
31	9	10	21	6	0	18	20	74	6	14	
963	4	0	2	231	8	12	3	105	9	12	
								472	6	19	

lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.
27	10	17	18	48	11	18	21				
17	9	12	14	42	10	14	0				
33	6	13	15	40	9	16	20				
10	11	13	15	36	8	15	22				
19	10	19	8	38	10	10	0				
16	0	10	23	53	0	17	13				
126	2	7	21	261	4	13	4				

oz.	dwt.	gr.		lb.	oz.	dwt.	gr.
74	19	23	Dishes and covers	56	9	2	0
64	14	17	Plates	69	10	0	0
74	19	11	Spoons and ladles.	25	0	19	0
66	13	9	Waiters	17	1	0	0
74	14	11	Salts and castors .	4	4	13	0
14	10	3	8 Candlesticks . .	14	0	0	0
19	11	14	Other articles . .	5	5	5	2
390	3	16	Total . .	192	6	19	2

AVOIRDUPOIS WEIGHT, p. 22.

cwt.	qr.	lb.	oz.	dr.	Ton	cwt.	qr.	lb.	oz.
14	1	25	14	9	15	12	1	10	10
13	2	20	1	15	71	8	2	6	0
9	3	6	7	3	83	19	3	15	5
10	0	18	12	11	36	7	0	20	14
7	2	27	3	2	47	11	1	27	11
6	1	19	8	1	63	5	2	19	7
4	3	10	15	5	12	13	1	14	9
12	2	9	9	13	9	7	0	5	10
79	2	26	8	11	340	5	2	8	2

COMPOUND ADDITION,

13

Cwt.	gr.	lb.	oz.	dr.	Cwt.	gr.	lb.	oz.
24	3	25	10	8	51	3	19	10
17	2	1	3	11	17	0	26	15
8	1	2	14	0	18	1	12	8
9	0	26	12	15	12	2	9	14
4	1	6	5	1	4	1	6	10
64	1	6	14	3	104	1	19	9

	Cwt.	gr.	lb.	oz.	dr.		Cwt.	gr.	lb.
Teas . . .	0	3	14	0	0	Soap . . .	2	1	14
Coffee. . .	0	1	23	0	0	White lead	2	3	26
Sugar . . .	3	2	5	0	0	Glue . . .	0	1	15
Spices. . .	0	2	3	13	9	Rock salt .	4	3	24
Hops . . .	13	1	24	0	0	Basket ditto	0	3	15
Other articles	3	0	17	7	13	Mustard .	1	2	27
Total weight	22	0	3	5	6		13	1	9

APOTHECARIES' WEIGHT.

lb.	oz.	dr.	sc.	lb.	oz.	dr.	sc.	dr.	sc.	gr.
14	6	3	2	17	6	5	2	7	5	17
11	2	7	1	33	9	1	0	6	2	13
15	11	5	0	15	8	6	2	0	1	16
19	5	4	1	82	11	2	1	7	0	1
18	0	2	1	94	6	7	2	2	2	5
17	11	7	2	15	10	5	1	8	1	19
16	10	6	2	260	5	4	2	34	2	11
114	1	5	0							

Sc.	gr.		lb.	oz.	dr.	sc.	gr.
1	17	First ingredient weighed	3	7	0	0	0
5	3	Second	0	11	7	0	13
7	8	Third	7	0	0	2	0
6	4	Fourth	1	0	3	1	0
9	12	Fifth	5	5	2	1	7
10	15	Total weight .	18	0	5	2	0
40	19						

WOOL WEIGHT.

<i>Sa.</i>	<i>wy.</i>	<i>td.</i>	<i>st.</i>	<i>Las.</i>	<i>wy.</i>	<i>td.</i>	<i>st.</i>	<i>Sa.</i>	<i>wy.</i>	<i>td.</i>
45	1	3	1	16	5	3	1	15	1	4
18	0	6	0	13	2	2	1	13	0	5
29	1	1	1	19	9	5	0	10	0	2
14	0	5	1	16	2	1	1	7	0	4
5	1	4	0	6	8	5	1	8	0	3
6	1	3	1	4	7	3	1	6	1	2
119	0	2	0	3	0	2	0	117	1	6
240	0	0	0	78	12	3 $\frac{1}{2}$	1	176	3	26

LONG MEASURE.

<i>Mi.</i>	<i>fur.</i>	<i>po.</i>	<i>yds.</i>	<i>Lea.</i>	<i>mi.</i>	<i>fur.</i>	<i>po.</i>	<i>Mi.</i>	<i>fur.</i>	<i>po.</i>
37	3	14	2	13	1	7	10	12	2	39
28	14	16	2	40	2	6	30	15	1	19
19	10	9	4	15	1	0	12	17	1	10
10	5	6	3	29	0	7	29	19	2	15
20	2	2	2	63	1	2	16	14	2	16
30	1	1	4	199	1	2	12	122	1	2
148	4	11	0 $\frac{1}{2}$	362	0	2	29	200	3	21

	<i>Mi.</i>	<i>fur.</i>	<i>po.</i>	<i>yd.</i>	<i>in.</i>
Distance from A to B	3	2	7	0	0
„ from B to C	17	0	13	0	0
„ from C to D	0	7	10	5	0
„ from D to E	5	0	33	1	7
Sum =	26	2	24	$\frac{1}{2}$	7
or = 26 2 24 2 1;					
since $\frac{1}{2}$ yard together with 7 inches = 25 inches = 2 feet 1 inch.					

CLOTH MEASURE.

<i>E.c.</i>	<i>qrs.</i>	<i>nl.</i>	<i>in.</i>	<i>Fr.c.</i>	<i>qrs.</i>	<i>nl.</i>	<i>Yds.</i>	<i>qrs.</i>	<i>nl.</i>	<i>in.</i>
97	2	2	1	126	4	2	785	2	3	1
58	1	3	2	233	5	3	392	3	2	2
20	4	2	1	87	1	2	86	1	1	0
9	3	0	2	32	3	1	7	0	2	1
0	4	3	1	25	2	0	0	3	1	2
0	2	2	2	16	0	2	0	0	2	1
188	0	0	0	521	5	2	1273	0	2	$\frac{1}{2}$

COMPOUND ADDITION.

15

<i>E.s.</i>	<i>qrs.</i>	<i>als.</i>
95	3	3
83	4	2
90	2	1
25	1	0
50	0	3
6	3	2
<hr/>		
352	0	3

	<i>E.s.</i>	<i>qrs.</i>	<i>als.</i>
First parcel contained	400	4	3
Second	976	3	0
Third.	612	2	1
Fourth	43	4	0
	<hr/>		
Sum =	2033	4	0
Or, =	2033	<i>E.s.</i>	1 <i>yd.</i>

SQUARE MEASURE.

<i>Ac.</i>	<i>p.</i>	<i>yd.</i>	<i>ft.</i>
2	1	28	6
3	30	10	7
1	38	30	3
0	18	0	2
1	0	12	0
1	20	13	3
<hr/>			
10	30	4½	3

<i>Ac.</i>	<i>rd.</i>	<i>p.</i>
409	1	86
81	3	20
94	2	10
8	0	17
0	3	39
0	0	25
<hr/>		
595	0	27

<i>Sq. ac.</i>	<i>rd.</i>	<i>p.</i>
4061	0	24
2731	2	3
841	3	19
96	2	39
85	0	10
40	1	0
<hr/>		
7856	2	15

	<i>Ac.</i>	<i>rd.</i>	<i>p.</i>	<i>yd.</i>	<i>ft.</i>
First piece of land contains	7	3	24	0	0
Second	18	1	16	0	0
Third	20	0	5	8	0
Fourth	15	0	0	24	7
	<hr/>				
Sum =	61	1	6	1½	7

CUBIC OR SOLID MEASURE.

<i>Yds.</i>	<i>feet</i>	<i>in.</i>
729	13	107
904	17	24
381	12	11
209	18	17
57	24	99
106	4	101
<hr/>		
2389	7	359

<i>Yds.</i>	<i>feet</i>	<i>in.</i>
159	22	7
76	19	28
24	13	12
16	2	8
102	26	107
29	10	1
<hr/>		
409	11	163

<i>Yds.</i>	<i>feet</i>	<i>in.</i>
56	13	10
127	26	59
38	4	5
37	5	7
29	20	505
17	19	207
<hr/>		
307	6	793

COMPOUND ADDITION.

WINE MEASURE.

<i>Tuns</i>	<i>pipe</i>	<i>hhd.</i>	<i>gal.</i>	<i>qt.</i>	<i>Tun.</i>	<i>pun.</i>	<i>tier.</i>	<i>gal.</i>
83	1	1	62	3	61	2	1	40
32	0	0	12	1	53	1	0	39
80	1	1	40	2	48	2	1	13
91	1	0	20	0	32	0	0	10
53	1	1	55	3	25	1	1	9
42	1	0	0	2	17	2	0	41
9	0	1	10	1	8	1	1	0
394	0	1	13	0	248	0	1	26

<i>Tuns</i>	<i>pun.</i>	<i>tier.</i>	<i>gal.</i>	<i>qt.</i>	<i>Hhd.</i>	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>
56	2	0	41	3	53	12	2	1
32	1	1	16	2	91	61	3	1
48	2	1	10	1	81	0	2	1
25	0	0	38	0	90	15	0	0
10	2	1	19	2	8	6	2	1
8	0	1	0	3	0	57	1	0
0	2	0	40	1	0	0	3	1
183	0	1	41	0	325	28	3	1

	<i>Tuns</i>	<i>pipe</i>	<i>hhd.</i>	<i>gal.</i>	<i>qts.</i>
Claret .	8	0	0	0	0
Port .	12	0	1	9	0
Sherry .	4	1	0	84	0
Lisbon .	0	0	3	12	3
Sum =	25	0	3	42	3

ALE AND BEER MEASURE.

<i>Hhds.</i>	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>	<i>Pun.</i>	<i>a. fr.</i>	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>	<i>Hds.</i>	<i>kil.</i>	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>
90	50	2	1	365	7	6	3	1	98	2	17	3	1
19	35	3	0	84	5	7	2	0	54	1	16	2	0
78	16	1	1	10	2	3	1	1	33	0	10	1	1
16	3	0	1	0	6	2	3	1	20	1	8	0	1
9	52	3	0	0	0	5	1	0	11	2	6	3	1
8	13	2	1	461	7	1	3	1	219	0	5	3	0
223	10	1	0										

	<i>Tuns.</i>	<i>hhd.</i>	<i>gal.</i>
Sent 1st time	0	5	56 = 5 hhd. 1 bar. 20 gal.
„ 2nd time	0	0	171 = 9 kil. 1 fir.
„ 3rd time	1	3	50 = 1 tun 3 hhd. 50 gal.
Sum =	4	1	7

COMPOUND SUBTRACTION.

17

DRY MEASURE.

<i>Last. wey</i>	<i>qrs.</i>	<i>coom.</i>	<i>Last. qrs.</i>	<i>bu.</i>	<i>pk.</i>	<i>Exported</i>	<i>last. qrs.</i>	<i>bu.</i>			
99	1	3	1	72	6	7	2	Wheat	18	2	5
65	1	2	0	37	9	6	3	Rye .	29	6	7
49	1	1	0	68	4	2	1	Beans .	15	9	3
120	0	1	1	38	3	0	2	Oats .	46	0	6
16	1	3	1	17	7	5	3	Sum =	109	9	5
352	0	1	1	235	1	6	3				

TIME.

<i>Year.</i>	<i>mon.</i>	<i>week</i>	<i>day</i>	<i>ho.</i>	<i>min.</i>
39	10	2	4	23	58
56	3	1	5	20	50
39	11	3	6	18	10
86	8	2	3	7	0
12	7	1	2	13	33
8	1	0	5	13	26
7	5	2	3	14	21
251	0	3	4	15	18

<i>Year</i>	<i>mon.</i>	<i>wk.</i>	<i>day.</i>	<i>Hour.</i>	<i>min.</i>	<i>sec.</i>
76	8	3	6	20	37	40
57	11	2	3	17	20	35
34	9	3	5	21	16	34
57	6	1	2	16	27	46
35	10	2	4	22	19	52
56	9	3	3	19	22	16
20	6	1	2	21	31	37
340	3	2	4	138	56	20

COMPOUND SUBTRACTION.

	<i>£.</i>	<i>s.</i>	<i>d.</i>		<i>£.</i>	<i>s.</i>	<i>d.</i>		<i>£.</i>	<i>s.</i>	<i>d.</i>
From	45	16	9½		8	12	10½		453	6	2½
Take	13	8	5½		5	16	9½		165	1	10½
	32	8	4½		2	16	1½		288	4	3½
From	15	7	10		284	9	8		2474	0	6½
Take	6	4	6		192	19	3		1972	17	7½
	9	3	5		91	10	5		501	2	10½

COMPOUND SUBTRACTION.

	<i>£.</i>	<i>s.</i>	<i>d.</i>
Received at first time	56	2	6
„ another time	32	17	5½
„ third time	101	6	2
	190	6	1½
	849	6	8½
Money due	659	0	7½

TROY WEIGHT.

<i>lb.</i>	<i>oz.</i>	<i>dwt.</i>	<i>gr.</i>	<i>lb.</i>	<i>oz.</i>	<i>dwt.</i>	<i>gr.</i>	<i>lb.</i>	<i>oz.</i>	<i>dwt.</i>	<i>gr.</i>
8	7	17	21	25	6	0	8	336	0	16	0
6	2	13	9	16	8	12	15	119	6	9	18
2	5	4	12	8	9	7	17	216	6	6	6
<i>lb.</i>	<i>oz.</i>	<i>dwt.</i>	<i>gr.</i>	<i>lb.</i>	<i>oz.</i>	<i>dwt.</i>	<i>gr.</i>	<i>lb.</i>	<i>oz.</i>	<i>dwt.</i>	<i>gr.</i>
637	9	0	8	8947	0	0	8				
288	1	9	20	5398	6	18	12				
349	7	10	12	3548	5	1	12				

AVOIRDUPOIS WEIGHT.

<i>Ton.</i>	<i>cwt.</i>	<i>qr.</i>	<i>lb.</i>	<i>oz.</i>	<i>Cwt.</i>	<i>qr.</i>	<i>lb.</i>	<i>oz.</i>	<i>dr.</i>
121	14	2	20	14	17	1	0	9	12
96	12	1	24	9	8	2	23	12	13
25	2	0	24	5	8	2	4	12	15
<i>lb.</i>	<i>oz.</i>	<i>dr.</i>			<i>Ton.</i>	<i>cwt.</i>	<i>qr.</i>	<i>lb.</i>	
56	2	0			Bought	2	5	1	7
13	9	11			Sold	1	19	0	20
42	8	5			Remainder	6	0	15	

APOTHECARIES' WEIGHT.

<i>lb.</i>	<i>oz.</i>	<i>dr.</i>	<i>sc.</i>	<i>gr.</i>	<i>lb.</i>	<i>oz.</i>	<i>dr.</i>	<i>sc.</i>	<i>gr.</i>	<i>lb.</i>	<i>oz.</i>	<i>dr.</i>	<i>sc.</i>	<i>gr.</i>
33	9	6	2	18	8	3	2	1	7	46	0	0	0	0
17	6	5	1	12	0	10	7	0	15	17	8	3	2	13
16	3	1	1	6	7	4	3	0	12	28	3	4	0	7

WOOL WEIGHT.

<i>La.</i>	<i>sa.</i>	<i>wy.</i>	<i>t.</i>	<i>st.</i>	<i>cl.</i>	<i>Wy.</i>	<i>t.</i>	<i>st.</i>	<i>cl.</i>	<i>lb.</i>
28	9	1	4	1	0	100	0	0	0	0
16	11	1	6	1	1	40	5	1	1	5
11	9	1	3½	1	1	59	½	0	0	2

COMPOUND SUBTRACTION.

19

<i>Sa.</i>	<i>wy.</i>	<i>t.</i>	<i>st.</i>
45	1	6	1
28	1	3	0
<hr/>			
17	0	3	1

<i>Sa.</i>	<i>wy.</i>	<i>t.</i>	<i>st.</i>
189	1	5	0
38	0	6	1
<hr/>			
151	0	4 $\frac{1}{2}$	1

LONG MEASURE.

<i>Lea.</i>	<i>mi.</i>	<i>fur.</i>	<i>po.</i>	<i>yd.</i>
160	1	3	20	2
84	2	6	28	3
<hr/>				
75	1	4	31	4 $\frac{1}{2}$

<i>Mi.</i>	<i>fur.</i>	<i>po.</i>	<i>yd.</i>	<i>ft.</i>	<i>in.</i>
70	7	13	1	1	2
20	0	14	2	2	8
<hr/>					
50	6	38	3 $\frac{1}{2}$	1	6

<i>Lea.</i>	<i>mi.</i>	<i>fur.</i>	<i>po.</i>	<i>yds.</i>
50	2	1	0	0
19	0	0	18	4
<hr/>				
31	2	0	21	1 $\frac{1}{2}$

<i>Mi.</i>	<i>fur.</i>	<i>po.</i>	<i>yd.</i>	<i>ft.</i>
79	4	0	0	0
12	6	0	3	2
<hr/>				
66	53	9	1 $\frac{1}{2}$	1

CLOTH MEASURE.

<i>Fl. c.</i>	<i>qr.</i>	<i>nl.</i>	<i>in.</i>
260	1	0	1
150	2	2	2
<hr/>			
109	1	1	1 $\frac{1}{2}$

<i>Yds.</i>	<i>qrs.</i>	<i>nl.</i>	<i>in.</i>
365	2	1	1
78	3	2	2
<hr/>			
286	2	2	1 $\frac{1}{2}$

<i>E. c.</i>	<i>qr.</i>	<i>nl.</i>
156	0	0
30	1	1
<hr/>		
125	3	3

<i>Fr. c.</i>	<i>qr.</i>	<i>nl.</i>
908	0	0
170	4	3
<hr/>		
737	1	1

<i>Yds.</i>	<i>qr.</i>	<i>na.</i>	<i>in.</i>
856	0	0	0
200	2	1	1
<hr/>			
655	1	2	1 $\frac{1}{2}$

<i>Yds.</i>	<i>qr.</i>	<i>na.</i>	<i>in.</i>
19	0	0	0
0	3	2	2
<hr/>			
18	0	1	1 $\frac{1}{2}$

SQUARE MEASURE.

<i>Ac.</i>	<i>rd.</i>	<i>po.</i>	<i>yd.</i>	<i>ft.</i>
69	2	13	14	7
30	3	28	30	4
<hr/>				
38	2	24	14 $\frac{1}{2}$	3

<i>Ac.</i>	<i>rd.</i>	<i>po.</i>	<i>yd.</i>	<i>ft.</i>
783	0	30	23	1
186	3	36	27	2
<hr/>				
596	0	33	25 $\frac{1}{2}$	8

<i>Ac.</i>	<i>rd.</i>	<i>po.</i>	<i>yd.</i>	<i>ft.</i>
576	0	0	0	0
238	2	30	3	
<hr/>				
337	1	9	27 $\frac{1}{2}$	

<i>Ac.</i>	<i>rd.</i>	<i>po.</i>
780	2	0
396	3	15
<hr/>		
383	2	25

<i>Ac.</i>	<i>rd.</i>	<i>po.</i>	<i>yd.</i>	<i>ft.</i>
800	0	0	0	0
100	2	0	0	8
<hr/>				
699	1	39	29 $\frac{1}{2}$	1

<i>Ac.</i>	<i>ro.</i>	<i>po.</i>	<i>yd.</i>	<i>ft.</i>
239	2	28	20	7
58	2	29	25	8
<hr/>				
180	3	38	24 $\frac{1}{2}$	8

WINE MEASURE.

<i>Tun.</i>	<i>pi.</i>	<i>hhd.</i>	<i>gal.</i>	<i>qt.</i>
200	1	0	30	1
156	1	1	48	2
<hr/>				
43	1	0	44	3

<i>Hhd.</i>	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>
367	20	2	0
148	38	3	1
<hr/>			
218	44	2	1

<i>Tun.</i>	<i>pun.</i>	<i>tier.</i>	<i>gal.</i>
209	1	1	25
131	2	1	38
<hr/>			
77	1	1	29

COMPOUND SUBTRACTION.

<i>Tun.</i>	<i>pt.</i>	<i>hhd.</i>	<i>gal.</i>	<i>qt.</i>
6	0	0	0	0
0	0	3	15	3
5	0	0	47	1

<i>Tun.</i>	<i>pun.</i>	<i>tier.</i>	<i>gal.</i>
28	1	0	0
15	0	1	19
13	0	0	23

ALE AND BEER MEASURE.

<i>Tun.</i>	<i>but.</i>	<i>hhd.</i>	<i>gal.</i>	<i>qt.</i>	<i>Hhd.</i>	<i>kil.</i>	<i>gal.</i>	<i>qts.</i>	<i>pt.</i>	<i>Pun.</i>	<i>fir.</i>	<i>gal.</i>	<i>qts.</i>	<i>pt.</i>
78	1	1	13	0	100	1	12	1	1	84	5	3	2	0
60	1	1	48	3	40	2	16	3	0	26	7	6	1	1
17	1	1	18	1	59	1	13	2	1	57	5	5	0	1

<i>Tun.</i>	<i>but.</i>	<i>hhd.</i>	<i>gal.</i>	<i>qt.</i>	<i>Pun.</i>	<i>hhd.</i>	<i>gal.</i>	<i>Pun.</i>	<i>hhd.</i>	<i>gal.</i>	<i>qts.</i>	<i>pt.</i>
12	1	0	0	0	19	1	0	27	1	48	2	0
8	0	0	50	3	10	1	40	24	1	49	2	1
4	0	1	3	1	8	$\frac{1}{2}$	14	2	$\frac{1}{2}$	52	3	1

8 Pun. 1 kil. 14 gall.

DRY MEASURE.

Wey.	qrs.	bus.	pk.	Wey.	qrs.	bus.	pk.	Last. wey.	qrs.	coom.	
12	1	3	2	100	3	2	3	65	0	3	1
8	4	2	3	86	4	5	2	46	1	3	1
3	2	0	3	13	3	5	1	18	1	0	0

<i>Qrs.</i>	<i>bus.</i>	<i>pk.</i>	<i>pot.</i>	<i>Wey.</i>	<i>qrs.</i>	<i>bus.</i>	<i>pk.</i>	<i>Loads.</i>	<i>qrs.</i>	<i>coom.</i>	<i>bus.</i>	<i>pk.</i>
79	1	1	0	20	0	0	0	8	2	1	0	0
34	2	1	3	8	3	0	2	0	4	0	3	2
44	6	3	1	11	1	7	2	7	3	0	0	2

TIME.

<i>Mon.</i>	<i>wk.</i>	<i>dys.</i>	<i>hrs.</i>	<i>min.</i>	<i>Years.</i>	<i>mon.</i>	<i>wk.</i>	<i>dys.</i>	<i>Mon.</i>	<i>wk.</i>	<i>dys.</i>	<i>hrs.</i>
12	1	2	14	12	1650	9	2	3	18	0	4	10
7	2	3	9	50	486	2	3	5	9	2	5	21
4	2	6	4	22	1164	6	2	5	8	1	5	13

<i>Years.</i>	<i>mon.</i>	<i>wk.</i>	<i>dys.</i>	<i>hrs.</i>	<i>min.</i>	<i>sec.</i>	<i>Mon.</i>	<i>wk.</i>	<i>dys.</i>	<i>hrs.</i>	<i>Wks.</i>	<i>dys.</i>	<i>hrs.</i>	<i>min.</i>	<i>sec.</i>
400	0	0	0	0	0	0	87	0	0	0	39	0	0	0	0
98	3	0	0	8	0	10	43	2	3	1	13	6	20	11	13
301	9	3	6	15	59	50	43	1	3	23	25	0	3	48	47

COMPOUND MULTIPLICATION.

(2)	(3)	(4)	(5)
$\begin{array}{r} \text{£. s. d.} \\ 0 \ 9 \ 6 \\ \quad 3 \\ \hline 1 \ 8 \ 6 \end{array}$	$\begin{array}{r} \text{£. s. d.} \\ 0 \ 1 \ 3 \\ \quad 5 \\ \hline 0 \ 6 \ 3 \end{array}$	$\begin{array}{r} \text{£. s. d.} \\ 2 \ 11 \ 5 \\ \quad 9 \\ \hline 23 \ 2 \ 9 \end{array}$	$\begin{array}{r} \text{£. s. d.} \\ 0 \ 19 \ 6\frac{1}{2} \\ \quad 12 \\ \hline 11 \ 14 \ 6 \end{array}$

CONTRACTION I.

(2)	(3)	(4)	(5)
$\begin{array}{r} \text{£. s. d.} \\ 1 \ 9 \ 4 \\ \quad 4 \\ \hline 5 \ 17 \ 4 \\ \quad 7 \\ \hline 41 \ 1 \ 4 \end{array}$	$\begin{array}{r} \text{£. s. d.} \\ 3 \ 15 \ 3\frac{1}{2} \\ \quad 5 \\ \hline 18 \ 16 \ 5\frac{1}{2} \\ \quad 7 \\ \hline 131 \ 15 \ 2\frac{1}{2} \end{array}$	$\begin{array}{r} \text{£. s. d.} \\ 2 \ 4 \ 6 \\ \quad 6 \\ \hline 13 \ 7 \ 0 \\ \quad 7 \\ \hline 93 \ 9 \ 0 \end{array}$	$\begin{array}{r} \text{£. s. d.} \\ 0 \ 9 \ 6 \\ \quad 8 \\ \hline 3 \ 16 \ 0 \\ \quad 8 \\ \hline 30 \ 8 \ 0 \end{array}$

(6)	(7)	(8)	(9)
$\begin{array}{r} \text{£. s. d.} \\ 2 \ 3 \ 4 \\ \quad 12 \\ \hline 26 \ 0 \ 0 \\ \quad 8 \\ \hline 208 \ 0 \ 0 \end{array}$	$\begin{array}{r} \text{£. s. d.} \\ 0 \ 9 \ 9\frac{1}{2} \\ \quad 12 \\ \hline 5 \ 17 \ 6 \\ \quad 10 \\ \hline 58 \ 15 \ 0 \end{array}$	$\begin{array}{r} \text{£. s. d.} \\ 0 \ 2 \ 4 \\ \quad 12 \\ \hline 1 \ 8 \ 0 \\ \quad 11 \\ \hline 15 \ 8 \ 0 \end{array}$	$\begin{array}{r} \text{£. s. d.} \\ 0 \ 13 \ 4 \\ \quad 12 \\ \hline 8 \ 0 \ 0 \\ \quad 12 \\ \hline 96 \ 0 \ 0 \end{array}$

(10)
$\begin{array}{r} \text{£. s. d.} \\ 0 \ 2 \ 2 \\ \quad 10 \\ \hline 1 \ 1 \ 8 \\ \quad 11 \\ \hline 11 \ 18 \ 4 \\ \quad 11 \\ \hline 131 \ 1 \ 8 \end{array}$
= price of 1 yard.
= price of 10 yards.
= price of 110 yards.
= price of 1210 yards.

CONTRACTION II.

$$\begin{array}{r}
 \text{(2)} \\
 \begin{array}{r}
 \text{£. s. d.} \\
 0 \ 1 \ 6\frac{1}{2} \times 1 \\
 \hline
 2 \\
 0 \ 3 \ 1 \\
 \hline
 11 \\
 1 \ 13 \ 11 \\
 0 \ 1 \ 6\frac{1}{2} \text{ add.} \\
 \hline
 1 \ 15 \ 5\frac{1}{2}
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{(3)} \\
 \begin{array}{r}
 \text{£. s. d.} \\
 0 \ 4 \ 7\frac{1}{2} \times 1 \\
 \hline
 5 \\
 1 \ 3 \ 0\frac{1}{2} \\
 \hline
 9 \\
 10 \ 7 \ 2\frac{1}{2} \\
 0 \ 4 \ 7\frac{1}{2} \text{ add.} \\
 \hline
 10 \ 11 \ 9\frac{1}{2}
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{(4)} \\
 \begin{array}{r}
 \text{£. s. d.} \\
 0 \ 7 \ 10 \times 1 \\
 \hline
 12 \\
 4 \ 14 \ 0 \\
 \hline
 5 \\
 23 \ 10 \ 0 \\
 0 \ 7 \ 10 \text{ sub.} \\
 \hline
 23 \ 2 \ 2
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{(5)} \\
 \begin{array}{r}
 \text{£. s. d.} \\
 0 \ 12 \ 2 \times 2 \\
 \hline
 12 \\
 7 \ 6 \ 0 \\
 \hline
 8 \\
 58 \ 8 \ 0 \\
 1 \ 4 \ 4 \text{ sub.} \\
 \hline
 57 \ 3 \ 8
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{(6)} \\
 \begin{array}{r}
 \text{£. s. d.} \\
 2 \ 2 \ 3 \times 3 \\
 \hline
 12 \\
 25 \ 7 \ 0 \\
 \hline
 10 \\
 253 \ 10 \ 0 \\
 6 \ 6 \ 9 \text{ sub.} \\
 \hline
 247 \ 3 \ 3
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{(7)} \\
 \begin{array}{r}
 \text{£. s. d.} \\
 0 \ 4 \ 6 \\
 \hline
 12 \\
 2 \ 14 \ 0 \\
 0 \ 2 \ 3 \text{ for } \frac{1}{2} \text{ lb.} \\
 \hline
 2 \ 16 \ 3
 \end{array}
 \end{array}$$

EXAMPLES IN WEIGHTS AND MEASURES.

$$\begin{array}{r}
 \text{lb. os. dr. sc. gr.} \\
 2 \ 4 \ 2 \ 1 \ 2 \\
 \hline
 7 \\
 16 \ 6 \ 0 \ 1 \ 14
 \end{array}$$

$$\begin{array}{r}
 \text{Cwt. gr. lb. os.} \\
 27 \ 1 \ 13 \ 12 \\
 \hline
 12 \\
 328 \ 1 \ 25 \ 0
 \end{array}$$

$$\begin{array}{r}
 \text{Ml. fur. po. yd.} \\
 24 \ 3 \ 20 \ 2 \\
 \hline
 6 \\
 146 \ 5 \ 2 \ 1
 \end{array}$$

$$\begin{array}{r}
 \text{Yds. qrs. na.} \\
 127 \ 2 \ 2 \\
 \hline
 8 \\
 1021 \ 0 \ 0
 \end{array}$$

$$\begin{array}{r}
 \text{Ac. rd. po.} \\
 27 \ 2 \ 1 \\
 \hline
 9 \\
 247 \ 2 \ 9
 \end{array}$$

$$\begin{array}{r}
 \text{Tuns hds. gal. pts.} \\
 25 \ 1 \ 20 \ 3 \\
 \hline
 5 \\
 126 \ 2 \ 38 \ 7
 \end{array}$$

$$\begin{array}{r}
 \text{Wys. qr. bu. pk.} \\
 27 \ 1 \ 7 \ 2 \\
 \hline
 7 \\
 191 \ 3 \ 4 \ 2
 \end{array}$$

$$\begin{array}{r}
 \text{Mon. wk. dys. hr. min.} \\
 175 \ 3 \ 6 \ 20 \ 59 \\
 \hline
 11 \\
 1935 \ 3 \ 5 \ 14 \ 49
 \end{array}$$

BILLS OF PARCELS.

A MERCER'S BILL.

	£.	s.	d.		£.	s.	d.
12 Yards of silk	at	0	15	2 per yard	9	2	0
9 Yards of flowered ditto	—	0	18	7½	8	7	7½
16 Yards of velvet	—	1	2	4	17	17	4
12 Yards of satin	—	0	13	9	8	5	0
27 Yards of brocade	—	0	15	7	21	0	9
13 Yards of lustring	—	0	6	3	4	1	3
				Sum	£68	13	11½

A STATIONER'S BILL.

5 Reams of paper	at	1	3	6 per rm.	5	17	6
112 Ditto	—	2	4	6	249	4	0
500 Dutch quills	—	0	3	9 per hd.	0	18	9
2000 Dutch common quills	—	0	2	3	2	5	0
19 Reams of fsep. paper	—	0	10	6 per rm.	9	19	6
				Sum	£268	4	9

A CARPENTER'S BILL.

64 Cubic feet of oak	at	4	3	per foot	13	12	0
125 Do. wrought and framed	—	5	8	—	35	8	4
19 Fir planks, each 20 ft. long	—	1	1	—	20	11	8
325 Feet of mahogany sashes	—	0	10	—	13	10	10
50 Ditto wainscot	—	0	9	—	1	17	6
10 Men's labour for 26 days	—	4	8	per day	60	13	4
				Sum	£145	13	8

A BRICKLAYER'S BILL.

1004 Feet of pointed old work	at	0	5½	per foot	23	0	2
1500 Grey stocks	—	4	6	per hd.	3	7	6
121 Pantiles	—	0	1½	each	0	15	7½
45 Hods of mortar	—	0	7½	—	1	8	1½
15 Bricklayers, 25 days	—	4	6	per day	84	7	6
10 Labourers, do.	—	3	0	—	37	10	0
55 Loads of rubbish carted away	2	6	per load	6	17	6	
				Sum	£157	6	5

PAINTER AND GLAZIER'S BILL.

	s.	d.	£.	s.	d.
1035 Yards of painting	at	0 7½	per yd.	32 6 10½	
39 Sash frames	—	0 11	each	1 15 9	
1265 Feet of best Newcastle glass	1	7½	per ft.	102 15 7½	
365 Feet, large size	—	2 1½	—	38 15 7½	
1000 Feet in lead work	—	1 1½	—	57 5 10.	
			Sum	£232 19 8½	

A WINE MERCHANT'S BILL.

12½ Dozen of white port	at	0 6 6	per bottle	48 15 0
3 Dozen of Lisbon	—	0 4 7	—	8 5 0
3 Dozen of claret	—	0 8 6	—	10 4 0
5 Dozen of Burgundy	—	0 11 6	—	34 10 0
7½ Dozen of Vidonia	—	0 3 10	—	17 5 0
8 Gallons brandy	—	1 2 6	per gall.	9 0 0
5 Gallons of rum	—	0 16 6	—	4 2 6
3 Gallons of gin	—	0 12 6	—	1 17 6
			Sum	£133 19 0

A GROCER'S BILL.

24½ lb. of green tea	at	12 6	per lb.	15 6 3
11½ lb. of Bohea	—	8 6	—	4 17 9
35½ lb. of best Hyson	—	13 10	—	24 14 6½
17 lb. of coffee	—	4 6	—	3 16 6
25 lb. of good loaf sugar	—	1 0½	—	1 6 0½
137 lb. of moist sugar	—	0 8½	—	4 17 0½
12 cakes of chocolate	—	4 6	per cake	2 14 0
19 lb. of raisins	—	1 2	per lb.	1 2 2
6½ lb. of cocoa	—	3 10	—	1 4 11
			Sum	£59 19 2½

COMPOUND DIVISION, p. 47.

$$\begin{array}{r}
 \text{(2)} \\
 3 \overline{) \begin{array}{ccc} \text{z.} & \text{s.} & \text{d.} \\ 751 & 14 & 7\frac{1}{2} \\ \underline{250} & 11 & 6\frac{1}{2} \end{array}}
 \end{array}
 \quad
 \begin{array}{r}
 \text{(3)} \\
 4 \overline{) \begin{array}{ccc} \text{z.} & \text{s.} & \text{d.} \\ 821 & 17 & 9\frac{1}{2} \\ \underline{205} & 9 & 5\frac{1}{2} \end{array}}
 \end{array}
 \quad
 \begin{array}{r}
 \text{(4)} \\
 5 \overline{) \begin{array}{ccc} \text{z.} & \text{s.} & \text{d.} \\ 2382 & 13 & 5\frac{1}{2} \\ \underline{476} & 10 & 8\frac{1}{2} \end{array}}
 \end{array}$$

$$\begin{array}{r}
 \text{(5)} \\
 6 \overline{) \begin{array}{ccc} \text{z.} & \text{s.} & \text{d.} \\ 28 & 2 & 1\frac{1}{2} \\ \underline{4} & 13 & 8\frac{1}{2} \end{array}}
 \end{array}
 \quad
 \begin{array}{r}
 \text{(6)} \\
 7 \overline{) \begin{array}{ccc} \text{z.} & \text{s.} & \text{d.} \\ 55 & 14 & 0\frac{1}{2} \\ \underline{7} & 19 & 1\frac{1}{2} \end{array}}
 \end{array}
 \quad
 \begin{array}{r}
 \text{(7)} \\
 8 \overline{) \begin{array}{ccc} \text{z.} & \text{s.} & \text{d.} \\ 6 & 5 & 4 \\ \underline{0} & 15 & 8 \end{array}}
 \end{array}$$

$$\begin{array}{r}
 \text{(8)} \\
 9 \overline{) \begin{array}{ccc} \text{z.} & \text{s.} & \text{d.} \\ 135 & 10 & 7 \\ \underline{15} & 1 & 2 \end{array}}
 \end{array}
 \quad
 \begin{array}{r}
 \text{(9)} \\
 10 \overline{) \begin{array}{ccc} \text{z.} & \text{s.} & \text{d.} \\ 21 & 18 & 4 \\ \underline{2} & 3 & 10 \end{array}}
 \end{array}
 \quad
 \begin{array}{r}
 \text{(10)} \\
 11 \overline{) \begin{array}{ccc} \text{z.} & \text{s.} & \text{d.} \\ 227 & 10 & 5 \\ \underline{20} & 13 & 8 \end{array}}
 \end{array}$$

$$\begin{array}{r}
 \text{(11)} \\
 12 \overline{) \begin{array}{ccc} \text{z.} & \text{s.} & \text{d.} \\ 1332 & 11 & 8\frac{1}{2} \\ \underline{111} & 0 & 11\frac{1}{2} \end{array}}
 \end{array}$$

CONTRACTION I., p. 48.

$$\begin{array}{r}
 \text{(2)} \\
 5 \overline{) \begin{array}{ccc} \text{z.} & \text{s.} & \text{d.} \\ 120 & 10 & 0 \\ \underline{24} & 2 & 0 \\ \underline{6} & 0 & 6 \end{array}}
 \end{array}
 \quad
 \begin{array}{r}
 \text{(3)} \\
 7 \overline{) \begin{array}{ccc} \text{z.} & \text{s.} & \text{d.} \\ 57 & 3 & 7 \\ \underline{8} & 3 & 4\frac{1}{2} \\ \underline{1} & 12 & 8 \end{array}}
 \end{array}
 \quad
 \begin{array}{r}
 \text{(4)} \\
 12 \overline{) \begin{array}{ccc} \text{z.} & \text{s.} & \text{d.} \\ 85 & 6 & 0 \\ \underline{7} & 2 & 2 \\ \underline{1} & 3 & 8\frac{1}{2} \end{array}}
 \end{array}$$

$$\begin{array}{r}
 \text{(5)} \\
 99 \left\{ \begin{array}{l} \text{(11)} \\ \text{(9)} \end{array} \right. \overline{) \begin{array}{ccc} \text{z.} & \text{s.} & \text{d.} \\ 31 & 2 & 10\frac{1}{2} \\ \underline{2} & 16 & 7\frac{1}{2} \\ \underline{0} & 6 & 3\frac{1}{2} \end{array}}
 \end{array}
 \quad
 \begin{array}{r}
 \text{(6)} \\
 112 \left\{ \begin{array}{l} \text{(4)} \\ \text{(4)} \\ \text{(7)} \end{array} \right. \overline{) \begin{array}{ccc} \text{z.} & \text{s.} & \text{d.} \\ 18 & 18 & 0 \\ \underline{4} & 14 & 6 \\ \underline{1} & 3 & 7\frac{1}{2} \\ \underline{0} & 3 & 4\frac{1}{2} \end{array}}
 \end{array}$$

EXAMPLES, p. 48.

$$\begin{array}{r}
 \text{(2)} \\
 \begin{array}{r}
 \begin{array}{ccccccc}
 \text{z.} & \text{s.} & \text{d.} & & \text{z.} & \text{s.} & \text{d.} \\
 37 &) & 23 & 15 & 7\frac{1}{2} & (& 0 & 12 & 10\frac{1}{2}
 \end{array} \\
 \hline
 20 \\
 475 \\
 444 \\
 \hline
 31 \\
 12 \\
 \hline
 379 \\
 370 \\
 \hline
 9 \\
 4 \\
 \hline
 37 \\
 37 \\
 \hline
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{(3)} \\
 \begin{array}{r}
 \begin{array}{ccccccc}
 \text{z.} & \text{s.} & \text{d.} & & \text{z.} & \text{s.} & \text{d.} \\
 53 &) & 199 & 3 & 10 & (& 3 & 15 & 2
 \end{array} \\
 \hline
 159 \\
 40 \\
 20 \\
 \hline
 803 \\
 53 \\
 \hline
 273 \\
 265 \\
 \hline
 8 \\
 12 \\
 \hline
 106 \\
 106 \\
 \hline
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{(4)} \\
 \begin{array}{r}
 \begin{array}{ccccccc}
 \text{z.} & \text{s.} & \text{d.} & & \text{z.} & \text{s.} & \text{d.} \\
 138 &) & 675 & 12 & 6 & (& 4 & 17 & 11
 \end{array} \\
 \hline
 552 \\
 123 \\
 20 \\
 \hline
 2472 \\
 138 \\
 \hline
 1092 \\
 966 \\
 \hline
 126 \\
 12 \\
 \hline
 1518 \\
 1518 \\
 \hline
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{(5)} \\
 \begin{array}{r}
 \begin{array}{ccccccc}
 \text{z.} & \text{s.} & \text{d.} & & \text{z.} & \text{s.} & \text{d.} \\
 365 &) & 315 & 3 & 10\frac{1}{2} & (& 0 & 17 & 3\frac{1}{2}
 \end{array} \\
 \hline
 20 \\
 6303 \\
 365 \\
 \hline
 2653 \\
 2555 \\
 \hline
 98 \\
 12 \\
 \hline
 1186 \\
 1095 \\
 \hline
 91 \\
 4 \\
 \hline
 365 \\
 365 \\
 \hline
 \end{array}
 \end{array}$$

EXAMPLES IN WEIGHTS AND MEASURES.

(1)

$$\begin{array}{r} \text{lbs.} \quad \text{oz.} \quad \text{dwt.} \quad \text{grs.} \\ 7 \text{) } 23 \quad 7 \quad 6 \quad 12 \\ \hline 3 \quad 4 \quad 9 \quad 12 \end{array}$$

(2)

$$\begin{array}{r} \text{lbs.} \quad \text{oz.} \quad \text{dr.} \quad \text{sc.} \quad \text{grs.} \\ 12 \text{) } 13 \quad 1 \quad 2 \quad 0 \quad 10 \\ \hline 1 \quad 1 \quad 0 \quad 2 \quad 10 \end{array}$$

(3)

$$\begin{array}{r} \text{Cwt.} \quad \text{qrs.} \quad \text{lbs.} \\ 28 \left\{ \begin{array}{l} 4 \text{) } 1061 \quad 2 \quad 0 \\ 7 \text{) } 265 \quad 1 \quad 14 \end{array} \right. \\ \hline 37 \quad 3 \quad 18 \end{array}$$

(Intercalary.)

$$\begin{array}{r} \text{Cwt.} \quad \text{qrs.} \quad \text{lbs.} \\ 144 \left\{ \begin{array}{l} 12 \text{) } 6314 \quad 1 \quad 12 \\ 12 \text{) } 526 \quad 0 \quad 22 \end{array} \right. \\ \hline 43 \quad 3 \quad 13\frac{1}{2} \end{array}$$

(4)

$$\begin{array}{r} \text{Mi.} \quad \text{fur.} \quad \text{po.} \quad \text{yd.} \quad \text{ft.} \quad \text{in.} \\ 39 \text{) } 375 \quad 2 \quad 7 \quad 2 \quad 1 \quad 2 \quad (9 \quad 4 \quad 39 \quad 0 \quad 2 \quad 8 \text{ Ans.} \\ \hline 351 \\ \hline 24 \\ \hline 8 \\ \hline 194 \\ \hline 156 \\ \hline 38 \\ \hline 40 \\ \hline 1527 \\ \hline 117 \\ \hline 357 \\ \hline 351 \\ \hline 6 \\ \hline 5\frac{1}{2} \\ \hline 35 \\ \hline 3 \\ \hline 106 \\ \hline 78 \\ \hline 28 \\ \hline 12 \\ \hline 338 \\ \hline 312 \\ \hline 26 \end{array}$$

(5)

$$\begin{array}{r} \text{Yds.} \quad \text{qrs.} \quad \text{nls.} \quad \text{Yds.} \quad \text{qrs.} \quad \text{nls.} \\ 47 \text{) } 571 \quad 2 \quad 1 \quad (12 \quad 0 \quad 2 \text{ Ans.} \\ \hline 564 \\ \hline 7 \\ \hline 4 \\ \hline 30 \\ \hline 4 \\ \hline 121 \\ \hline 94 \\ \hline 27 \end{array}$$

(6)

$$\begin{array}{r} \text{Ac.} \quad \text{ro.} \quad \text{po.} \quad \text{Ac.} \quad \text{ro.} \quad \text{po.} \\ 51 \text{) } 51 \quad 2 \quad 3 \quad (1 \quad 0 \quad 1 \text{ Ans.} \\ \hline 51 \\ \hline 2 \\ \hline 40 \\ \hline 83 \\ \hline 51 \\ \hline 32 \end{array}$$

REDUCTION, p. 56.

(2)
In 12*l.*, how many farthings?

$$\begin{array}{r}
 \text{£.} \\
 12 \\
 \underline{20} \\
 240\text{s.} \\
 12 \\
 \underline{2880d.} \\
 4 \\
 11520 \text{ far.}
 \end{array}$$

(4)
In 35 guineas, how many farthings?

$$\begin{array}{r}
 35 \text{ guineas.} \\
 21 \\
 \underline{35} \\
 70 \\
 \underline{735\text{s.}} \\
 12 \\
 \underline{8820d.} \\
 4 \\
 35280 \text{ far.}
 \end{array}$$

(6)
In 231*l.* 16*s.*, how many ducats at 4*s.* 9*d.* each?

<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>s.</i>
4	9	231	16
12		20	
57 <i>d.</i>		4636 <i>s.</i>	
		12 ducats.	
57)		55632 <i>d.</i> (976	
		513	
		433	
		399	
		342	
		342	

(3)
In 6169 pence, how many pounds?

$$\begin{array}{r}
 12 \) \ 6169d. \\
 2,0 \) \underline{51,4\text{s. } 1d.} \\
 \underline{25\text{l. } 14\text{s. } 1d.}
 \end{array}$$

(5)
In 420 quarter-guineas, how many moidores?

$$\begin{array}{r}
 4 \) \ 420 \text{ quarter-guineas.} \\
 \underline{105 \text{ guineas.}} \\
 21
 \end{array}$$

$$\begin{array}{r}
 105 \\
 \underline{210} \\
 27 \) \ 2205\text{s.} \ (\ 81 \text{ moi. } 18\text{s.} \\
 \underline{216} \\
 45 \\
 \underline{27} \\
 18
 \end{array}$$

(7)
In 274 marks and 87 nobles, how many pounds?

<i>marks.</i>	<i>nobles.</i>
274.	87
	2
3)	635 nobles.
	<u>211<i>l.</i> 13<i>s.</i> 4<i>d.</i></u>

NOTE.—2 nobles are equal to 1 mark, and 3 nobles equal to 1 pound.

(8)

In 1776 quarter-guineas,
how many sixpences?

$$\begin{array}{r}
 2 \text{) } 1776 \text{ qr.-guineas.} \\
 \underline{888} \text{ half-guineas.} \\
 21 \\
 \underline{888} \\
 1776 \\
 \underline{18648} \text{ sixpences.}
 \end{array}$$

(10)

In 50807 moidores, how
many pieces of coin each
4s. 6d.?

$$\begin{array}{r}
 \text{s. d.} \quad 50807 \text{ moi.} \\
 4 \quad 6 \quad \underline{27} \\
 2 \quad \underline{355649} \\
 9 \text{ sixp.} \quad \underline{101614} \\
 1371789\text{s.} \\
 \underline{2} \\
 9 \text{) } 2743578 \text{ sixpences.} \\
 \underline{304842} \text{ coins.}
 \end{array}$$

(12)

In 59 lbs. 13 dwts. 5 grs.,
how many grains?

$$\begin{array}{r}
 \text{lbs. dwts. grs.} \\
 59 \quad 13 \quad 5 \\
 \underline{12} \\
 708 \text{ oz.} \\
 \underline{20} \\
 14173 \text{ dwts.} \\
 \underline{24} \\
 56697 \\
 \underline{28346} \\
 340157 \text{ grs.}
 \end{array}$$

(9)

Reduce 1776 six-and-thirties
to half-crowns.

$$\begin{array}{r}
 1776 \\
 \underline{36} \\
 10656 \\
 \underline{5328} \\
 63936\text{s.} \\
 \underline{2}
 \end{array}$$

$$\begin{array}{r}
 5 \text{) } 127872 \text{ sixpences.} \\
 \underline{25574\frac{1}{2}} \text{ half-crowns.}
 \end{array}$$

(11)

In 213210 grains, how
many lbs.?

$$\begin{array}{r}
 24 \left\{ \begin{array}{l} 2 \text{) } 213210 \text{ grs.} \\ 12 \text{) } \underline{106605} \end{array} \right. \\
 2,0 \text{) } \underline{888,3} \text{ dwt. 18 gr.} \\
 12 \text{) } \underline{444} \text{ oz. 3 dwts.} \\
 \underline{37} \text{ lb. 3 dwt. 18 gr.}
 \end{array}$$

(13)

In 8012131 grains, how
many pounds?

$$\begin{array}{r}
 24 \left\{ \begin{array}{l} 4 \text{) } 8012131 \text{ grs} \\ 6 \text{) } \underline{2003032} - 3 \end{array} \right. \\
 2,0 \text{) } \underline{33383,8} \text{ dwt. 19 gr.} \\
 12 \text{) } \underline{16691} \text{ oz. 18 dwt.} \\
 \underline{1390} \text{ lbs. 11 oz.} \\
 [18 \text{ dwt. 19 gr.}]
 \end{array}$$

(14)

In 35tons, 17cwt. 1qr. 23lb. 7oz. 13dr., how many drams?

<i>Tons</i>	<i>cwts.</i>	<i>qrs.</i>	<i>lbs.</i>	<i>oz.</i>	<i>dr.</i>
35	17	1	23	7	13
<hr/>					
			20		
<hr/>					
			717		
<hr/>					
			4		
<hr/>					
			2869		
<hr/>					
			28		
<hr/>					
			22975		
<hr/>					
			5738		
<hr/>					
			80355		
<hr/>					
			16		
<hr/>					
			482137		
<hr/>					
			80355		
<hr/>					
			1285687		
<hr/>					
			16		
<hr/>					
			7714135		
<hr/>					
			1285687		
<hr/>					
			20571005		

(15)

In 37 cwt. 2 qrs. 17 lbs., how many lbs. troy; supposing a lb. avoirdupois to be equivalent to 14 oz. 11 dwts. 16 grs. troy?

<i>Cwt.</i>	<i>qrs.</i>	<i>lbs.</i>	<i>oz.</i>	<i>dwt.</i>	<i>grs.</i>
37	2	17	14	11	16
<hr/>					
		4			20
<hr/>					
		150			291
<hr/>					
		28			24
<hr/>					
		1217			1180
<hr/>					
		300			582
<hr/>					
		4217			7000
<hr/>					
		7000			
<hr/>					
		29519000			
<hr/>					
		7379750			
<hr/>					
		122995,8			
<hr/>					
		61497			
<hr/>					
		5124			

7000 grs. troy. = 1 lb. av.

$24 \left\{ \begin{array}{l} 4 \\ 6 \end{array} \right.$

$29519000 \text{ grs. troy.}$
 7379750
 $122995,8 \text{ dwts. 8 grs.}$
 $61497 \text{ oz. 18 dwts.}$
 $5124 \text{ lbs. 9 oz. 18 dwts. 8 grs.}$

(16)

How many barleycorns will reach round the world supposing it to be 8340 leagues?

$$\begin{array}{r}
 8340 \text{ leagues.} \\
 \underline{3} \\
 25020 \text{ miles.} \\
 \underline{8} \\
 200160 \text{ furlongs.} \\
 \underline{40} \\
 2) 8006400 \text{ poles.} \\
 \underline{5\frac{1}{2}} \\
 4003200 \\
 40032000 \\
 \underline{44035200} \text{ yards.} \\
 \underline{3} \\
 132105600 \text{ feet.} \\
 \underline{12} \\
 1585267200 \text{ inches.} \\
 \underline{3} \\
 \underline{4755801600} \text{ barleycorns.}
 \end{array}$$

(17)

In 17 pieces of cloth, each 27 Flemish ells, how many yards?

$$\begin{array}{r}
 27 \\
 17 \\
 \underline{189} \\
 27 \\
 \underline{459} \text{ Fl. ells.} \\
 \underline{3} \\
 4) 1377 \text{ qrs.} \\
 \underline{344} \text{ yds. 1 qr.}
 \end{array}$$

(18)

How many hours from the birth of Christ to Christmas, 1859, supposing the year to consist of 365 days 5 hours?

<i>days</i>	<i>hrs.</i>
365	5
<u>24</u>	
1465	
<u>730</u>	
8765 hrs. in 1 year.	
<u>1859</u>	
78885	
<u>43825</u>	
70120	
<u>8765</u>	
<u>16294135</u> hrs. in 1859 yrs.	

(19)

How many seconds are there in a solar year, which consists of 365 days, 5 hours, 48 minutes, 58 seconds?

<i>Da.</i>	<i>ho.</i>	<i>min.</i>	<i>sec.</i>
365	5	48	58
<u>24</u>			
8765			
<u>60</u>			
525948			
<u>60</u>			
<u>31556938</u> Ans.			

(20)

How long would it require to count 900 millions of pounds, at the rate of 100*l.* a minute, without intermission?

$$\begin{array}{r}
 100 \overline{) 900000000} \\
 6,0 \overline{) 900000,0} \text{ minutes.} \\
 24 \overline{) 150000} \text{ hours.} \\
 7 \overline{) 6250} \text{ days.} \\
 4 \overline{) 892} \text{ weeks 6 days.} \\
 13 \overline{) 223} \text{ months 6 days.} \\
 \underline{17 \text{ yrs. 2 mths. 6 days.}}
 \end{array}$$

(21)

If a person step $2\frac{1}{2}$ feet at a time, how many steps will he take in walking 18 miles?

$$\begin{array}{r}
 18 \text{ miles} = 95040 \text{ feet} = 380160 \text{ quarter-feet.} \\
 \therefore 9 \overline{) 380160} \\
 \underline{42240} = \text{the number of steps required.}
 \end{array}$$

(22)

How many times will a coach-wheel, of $17\frac{1}{2}$ feet in circumference, turn round in going from London to York, the distance being 199 miles?

$$\begin{array}{r}
 17\frac{1}{2} \text{ feet} = 35 \text{ half-feet} \\
 199 \text{ miles} = 2101440 \text{ half-feet.} \\
 35 \left\{ \begin{array}{l} 5 \overline{) 2101440} \\ 7 \overline{) 420288} \end{array} \right. \\
 \underline{60041\frac{1}{2}} \text{ times.}
 \end{array}$$

(23)

If a snail creep 2 feet 7 inches up a pole in the night-time, and slip down 18 inches during each day, how long a time will it take to reach the top of the pole, its height being 18 yards 11 inches?

The snail begins to ascend at the close of day, and creeps 31 inches up the pole the first night, and descends 18 inches the first day; therefore the snail is only 13 inches up the pole at the close of the first day. Now the whole height of the pole is 18 yards 11 inches, or 659 inches; hence at the close of the 49th day the snail will have ascended

$$49 \times 13 = 637 \text{ inches.}$$

As there are only 22 inches more to accomplish, the snail will gain the top on the 50th night.

(24)

An apprentice having been bound for 7 years, has served 3 years, 3 months, 3 weeks, and 3 days; how long has he yet to serve?

<i>Yr.</i>	<i>mon.</i>	<i>wk.</i>	<i>day.</i>	
7	0	0	0	
3	3	3	3	
<hr/>				
3	9	0	4	time he has yet to serve.

(25)

A horse, together with his harness, is worth 50 guineas; the harness is worth 15*l.* 11*s.* What is the value of the horse?

Horse and harness are worth 52*l.* 10*s.*

Harness is worth 15 11

Value of the horse is 36 19

(26)

The great bell at Christ Church, Oxford, weighs 7 tons, 11 cwt. 3 qrs. 4 lb., or 17000 lb.; that at St. Paul's weighs 5 tons, 2 cwt. 1 qr. 22 lb., or 11474 lb. And both put together weigh 28474 lb. The bell at Moscow weighs 198 tons, or 443520 lb. Divide 443520 by 28474, and the quotient is 15 bells; and 16410 lb. over, which reduced is equal to 7 tons, 6 cwt. 2 qrs. 2 lbs.

(Operation at length.)

Tons	cwt.	qrs.	lbs.
7	11	3	4
5	2	1	22
12	14	0	26
20			
254	cwt.		
4			
1016	qrs.		
28			
8154			
2032			
28474	lbs.		

Tons.
198
20
3960 cwt.
4
15840 qrs.
28
126720
31680
443520 lbs.

∴ 28474) 443520 (15 bells.

28474

158780

142370

28 ⁽⁴⁾) 16410 lbs. over.

⁽⁷⁾) 4102 - 2

4) 586 qrs. 2 lbs.

2,0) 14,6 cwt. 2 qrs.

7 tons, 6 cwt. 2 qrs. 2 lbs.

THE RULE OF THREE.

(3)

What is the value of a cwt. of sugar at $5\frac{1}{2}d.$ per lb.?

As 1 : 112 :: $5\frac{1}{2}$

$5\frac{1}{2}$

560

$\frac{1}{2}$ 56

12) 616d.

2,0) 5,1s. 4d.

2l. 11s. 4d. Ans.

(4)

What is the value of a chaldron of coals at $11\frac{1}{2}d.$ per bushel?

As 1 bush. : 36 bush. :: $11\frac{1}{2}d.$

$11\frac{1}{2}$

18

396

12) 414

2,0) 3,4s. 6d.

1l. 14s. 6d. Ans.

(5)

At $10\frac{1}{2}d.$ per pound, what is the value of a firkin of butter containing 56 lb.?

As 1 lb. : 56 lb. :: $10\frac{1}{2}d.$

$$\begin{array}{r}
 10\frac{1}{2} \\
 \hline
 28 \\
 560 \\
 12 \overline{) 588d.} \\
 2,0 \overline{) 4,9} \\
 \hline
 2l. \ 9s. \ Ans.
 \end{array}$$

(7)

At 3l. 9s. per cwt., what is the value of a pack of wool weighing 2 cwt. 2 qrs. 13 lb.?

Cwt.	Cwt. gr.	lb.	l.	s.
As 1	: 2	2	13	:: 3
	112	4		20
	112 lb.	10 qrs		69
		28		
		293 lbs.		
		69		
		2637		
		1758		
112)	20217	(180s. 6d., or 9l.		
	112	[0s. $6\frac{1}{11}\frac{1}{2}d.$ *]		
	901			
	896			
	57			
	12			
	684			
	672			
	12			

(6)

What is the value of a pipe of wine at $10\frac{1}{2}d.$ per pint?

As 1 pint : 1 pipe :: $10\frac{1}{2}d.$

$$\begin{array}{r}
 2 \\
 2 \text{ hhd.} \\
 \hline
 63 \\
 126 \text{ gal.} \\
 \hline
 8 \\
 1008 \text{ pints.} \\
 10\frac{1}{2} \\
 \hline
 10080 \\
 504 \\
 12 \overline{) 10584} \\
 2,0 \overline{) 88,2} \\
 \hline
 44l. \ 2s. \ Ans.
 \end{array}$$

(8)

What is the value of $1\frac{1}{2}$ cwt. of coffee at $5\frac{1}{2}d.$ per ounce?

os.	cwt.	gr.	d.
As 1	: 1	2	:: 5
		4	
		6 qrs.	
		28	
		168 lbs.	
		16	
		2688 oz.	
		5	
		13440	
		1344	
12)	14784		
2,0)	123,2		
	61l. 12s.		Ans.

* The student ought to be allowed to pass by the consideration of these fractional parts, until he shall have studied the nature of fractions.

(9)

What is the value of
19½ chaldrons of coals at
17. 11s. 6d. per chaldron?

<i>chald.</i>	<i>chald.</i>	<i>z.</i>	<i>s.</i>	<i>d.</i>
As 1 :	19½ :	1	11	6
		20		
		<u>31</u>		
		12		
		<u>378</u>		
		19½		
		<u>7182</u>		
		189		
12)	7371			
2,0)	61,4 3			
	<u>30l. 14s. 3d.</u>			

(10)

Bought 3 casks of raisins,
each weighing 2 cwt. 2 qrs.
25 lb.: what will they come
to at 2l. 1s. 8d. per cwt.?

<i>cwt.</i>	<i>cwt.</i>	<i>qr.</i>	<i>lb.</i>	<i>z.</i>	<i>s.</i>	<i>d.</i>
As 1 :	8	0	19 :	2	1	8
	112	4		20		
	<u>112</u>	32		<u>41</u>		
		28		12		
		<u>275</u>		<u>500d.</u>		
		64				
		<u>915</u>				
		500				
112)	457500					
	<u>448</u>					
	950	12)	4084½d.			
	<u>896</u>	2,0)	34,0s. 4½d.			
			<u>17l. 0s. 4½d.</u>			
			540			
			<u>448</u>			
			92			
			<u>4</u>			
			368			
			<u>336</u>			
			32			

(11)

What is the value of
2 qrs. 1 nail of velvet at
19s. 8½d. per E. ell?

<i>qr.</i>	<i>qr.</i>	<i>nl.</i>	<i>s.</i>	<i>d.</i>
As 5 :	2	1 :	19	8½
	4	4	12	
	<u>20nls.</u>	<u>9nls.</u>	<u>236</u>	
			4	
			<u>946</u>	
			9	
	2,0)	851,4		
	4)	425 ½d.		
	12)	106 ½		
		<u>8s. 10½d. ¼d.</u>		

(12)

Bought 12 pockets of hops,
each weighing 1 cwt. 2 qrs.
17 lb.: what do they come
to at 4l. 1s. 4d. per cwt.?

<i>cwt.</i>	<i>cwt.</i>	<i>qr.</i>	<i>lb.</i>	<i>z.</i>	<i>s.</i>	<i>d.</i>
As 1 :	1	2	17 :	4	1	4
	112		12	20		
	<u>112 lbs.</u>	<u>19</u>	<u>3</u>	<u>8</u>	<u>81s.</u>	
		4		12		
		<u>79</u>		<u>976d.</u>		
		28				
		<u>640</u>				
		158				
		<u>2220 lbs.</u>				

Multiply 976 and 2220
together; their product is
2166720, which being di-
vided by 112 gives a quo-
tient 19345½d. $\frac{1}{4}$, or 80l.
12s. 1½d. $\frac{1}{4}$, the Ans. re-
quired.

(13)

What is the tax on
745*l.* 14*s.* 8*d.* at 3*s.* 6*d.* in
the pound?

<i>£.</i>	<i>£.</i>	<i>s.</i>	<i>d.</i>	<i>s.</i>	<i>d.</i>
As 1 :	745	14	8	::	3 6
240		20			12
<u>240<i>d.</i></u>	149	14 <i>s.</i>			<u>42<i>d.</i></u>
		12			
	1789	76 <i>d.</i>			
		42			
	357	952			
	715	904			
240)	751	6992	(313	20½ <i>d.</i>
	720				
	316				
	240				
	<u>769</u>				
	720				
	499				
	480				
	192				
	4				
	768				
	720				
	<u>48</u>				

12)	31320½ <i>d.</i>
2,0)	261,0 <i>s.</i>
	<u>130<i>l.</i> 10<i>s.</i> 0½<i>d.</i> ¾</u>

(14)

If ¼ of a yard of velvet
cost 7*s.* 3*d.*, how many yards
can I buy for 13*l.* 15*s.* 6*d.*?

<i>s.</i>	<i>d.</i>	<i>£.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>
As 7	3 :	13	15	6	:: 3
12			20		
<u>87</u>			275		
			12		
			3306		
			3		
87)	9918	(114	qrs.	
	87				
	<u>121</u>				
	87				
	<u>348</u>				
	348				

4)	114	qrs.
	<u>28</u>	yds. 2 qrs. Ans.

(15)

If an ingot of gold, weigh-
ing 9 lb. 9 oz. 12 dwts. be
worth 411*l.* 12*s.*, what is the
cost per grain?

<i>lb.</i>	<i>oz.</i>	<i>dwt.</i>	<i>gr.</i>	<i>£.</i>	<i>s.</i>
As 9	9	12 :	1 ::	411	12
12				20	
<u>117</u>				8232	
20				12	
<u>2352</u>				98784	
24				4	
9408	56448	395136	(7, or		
4704		395136	1½ <i>d.</i>		
<u>56448</u>			per gr.		

(16)

How many quarters of corn can I buy for 40 guineas, at 4s. per bushel?

$$\begin{array}{r}
 \begin{array}{ccc} s. & \text{£} & \text{bush.} \\ \text{As } 4 & : 42 & :: 1 \end{array} \\
 \hline
 & & 20 \\
 4 & \overline{) 840} & \\
 8 & \overline{) 210} & \text{bushels.} \\
 \hline
 & & 26 \text{ qrs. 2 bush.}
 \end{array}$$

(18)

What is the value of a pack of wool weighing 2 cwt. 1 qr. 19 lb., at 8s. 6d. per stone?

$$\begin{array}{r}
 \begin{array}{cccc} st. & cwt. & qr. & lb. \\ \text{As } 1 & : 2 & 1 & 19 \end{array} \quad \begin{array}{cc} s. & d. \\ & : 8 & 6 \end{array} \\
 \hline
 14 & & 4 & & 12 \\
 \hline
 14 \text{ lbs.} & 9 \text{ qrs.} & & & 102d. \\
 \hline
 & & 28 & & \\
 & & 271 \text{ lbs.} & & \\
 & & 102 & & \\
 14 & \overline{) 27642} & & & \\
 12 & \overline{) 1974\frac{1}{2}d.} & \frac{1}{4} & & \\
 2,0 & \overline{) 16,4s.} & 6d. & & \\
 \hline
 & & 8l. & 4s. & 6\frac{1}{2}d. \quad \frac{1}{4} \text{ Ans.}
 \end{array}$$

(17)

If 1 E. ell 2 qrs. cost 4s. 7d., what will 39½ yards cost?

$$\begin{array}{r}
 \begin{array}{ccc} qrs. & qrs. & d. \\ \text{As } 7 & : 158 & :: 55 \end{array} \\
 \hline
 & & 55 \\
 & & \overline{790} \\
 & & 790 \\
 7 & \overline{) 8690} & \\
 12 & \overline{) 1241\frac{1}{2}d.} & \frac{1}{4} \\
 2,0 & \overline{) 10,3s.} & 5\frac{1}{2}d. \quad \frac{1}{4} \\
 \hline
 & & 5l. & 3s. & 5\frac{1}{2}d. \quad \frac{1}{4}
 \end{array}$$

(19)

Bought 4 bales of cloth, each containing 6 pieces, and each piece 27 yards, at 16l. 4s. per piece: what is the value of the whole and the rate per yard?

$$\begin{array}{r}
 \begin{array}{cccc} p. & ps. & \text{£} & s. \\ \text{As } 1 & : 6 & :: 16 & 4 \end{array} \\
 \hline
 & & 20 & \\
 & & 324 & \\
 & & 6 & \\
 2,0 & \overline{) 194,4} & & \\
 \hline
 & & 97l. & 4s. = \text{price of} \\
 & & 4 & [6 \text{ pieces.} \\
 & & 388l. & 16s. = \text{price of} \\
 & & \text{[the 4 bales, or 648 yds.}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{ccc} yds. & yds. & \text{£} & s. \\ \text{Hence } 648 & : 1 & :: 388 & 16 \end{array} \\
 \hline
 & & 20 & \\
 648 & \overline{) 7776} & &
 \end{array}$$

12s. the
[price per yd.

(20)

If an ounce of silver be worth 5s. 6d., what is the price of a cup that weighs 1 lb. 10 oz. 10 dwts. 4 grs.?

os.	lb. oz. dwt. gr.	s. d.
As 1	: 1 10 10 4 :: 5 6	
480	12	12
480	grs. 22	66d.
	20	
	450	
	24	
	1804	
	900	
	10804	
	66	
	64824	
	64824	
480)	713064	(6l. 3s. 9½d. ¾gr.

(22)

What is the half-year's rent of 547 acres of land at 15s. 6d. per acre?

a.	a.	s.	d.
As 1	: 547 :: 15 6		
	12		
	186d.		
	547		
	1302		
	744		
	930		
12)	101742	pence.	
2,0)	847,8s.6d.	[lyrent	
2)	423 18 6	the year-	
	211l. 19s. 3d.	half-	
		[yearly rent.	

(21)

What does 59 cwt. 2 qrs. 24 lb. of tobacco come to at 2l. 14s. 5d. per cwt.?

cwt.	cwt. qr. lb.	£.	s.	d.
As 1	: 59 2 24 :: 2 14 5			
112	4		20	
112 lb.	238 qr.		54s.	
	28		12	
	1928		653d.	
	476			
	6688 lb.			
	653			
112)	4367264	pence.		
	162l. 9s. 5½d. 11½gr.	Ans.		

(23)

Bought 1000 Flemish ells of cloth for 90l., how must I sell it per ell English to gain 10l. by the whole?

Fl. ells.	Eng. ell.	£.
As 1000	: 1 :: 100	
3	5	20
3000 qrs. 5 qrs.	2000s.	
	5	
3,000)	10,000	
	3s. 4d. per	
	[ell Eng.	

(24)

Suppose a gentleman's income is 500 guineas a year, and he spends 19s. 7d. per day, one day with another: how much will he have saved at the year's end?

<i>day.</i>	<i>days.</i>	<i>s.</i>	<i>d.</i>
As 1 :	365 ::	19	7
		<u>12</u>	
		235d.	
		<u>365</u>	
	12)	85775	
	2,0)	714,7	11
Yearly expense = 357 7 11			
,, income = 525			
Saved £167 12 1			

(25)

If $1\frac{1}{4}$ oz. of silver plate cost 10s. 11½d., what will a service weighing 327 oz. 12 dwts. 9 gr. cost at the same rate?

<i>oz. dwt.</i>	<i>oz. dwt. gr.</i>	<i>s. d.</i>
As 1 15 :	327 12 9 ::	10 11½
<u>20</u>	<u>20</u>	<u>12</u>
35	6552	131d.
<u>24</u>	<u>24</u>	<u>4</u>
gr.840	157257 gr.	525 f.
	525	
840)	82559925	
4)	98285½½½ f.	
12)	24571½d.	
2,0)	204,7s. 7d.	
	<u>102l. 7s. 7½d. ½½½</u>	

(26)

At 13s. 2½d. per yard, what is the price of a piece of cloth containing 52½ English ells?

<i>grs.</i>	<i>E. ells.</i>	<i>s. d.</i>
As 4 :	52½ ::	13 2½
<u>4</u>	<u>20 (nls. in 1 E. ell.)</u>	<u>12</u>
16 nls.	1055 nls.	158d.
	<u>634</u>	<u>4</u>
16)	668870	634 f.
4)	41804½ f.	
12)	10451d.	
2,0)	87,0s. 11d.	
	<u>43l. 10s. 11d. ½ Ans.</u>	

(27)

How many English ells of holland may be bought for 100 guineas at 8s. 9½d. per yard?

s.	d.		guineas.		gr.
As 8	9½	∴	100	∴ ∴	4
12			21		
105			2100		
4			12		
422 f.			25200		
			4		
			100800 f.		
			4		
			422) 403200		
			5) 955 qrs. 1 nl. ⅔		
			191 E. ells. 0 qr. 1 nl. ⅔		

(28)

What is the value of 172 pigs of lead, each weighing 3 cwt. 2 qrs. 17½ lbs., at 8l. 17s. 6d. per fother of 19½ cwt.?

cwt.		cwt.	qr.	lb.		l.	s.	d.
As 19½	∴	3	2	17½	∴ ∴	8	17	6
4		4				20		
78		14				177		
28		28				12		
624		129				2130		
156		28				819		
2184		409				19170		
2		2				2130		
4368 half-lbs.		819 half-lbs.				17040		
						1744470d.		
						172		
						4368) 300048840		
						12) 68692½d.		
						2,0) 572,4 4½		
						286l. 4s. 4½d.		

(29)

Bought 25 pieces of holland, each containing 25 English ells, for 300 guineas: what is that per yard?

$$\begin{array}{rcl}
 \text{As } 3125 \text{ } ^{\text{gr.}} : 4 \text{ } ^{\text{gr.}} :: 315 \text{ } ^{\text{s.}} \\
 \hline
 20 \\
 6300\text{s.} \\
 \hline
 4 \\
 3125 \overline{) 25200} \text{ (8s. 0}\frac{1}{2}\text{d. } \frac{11}{16}\frac{1}{2} \text{ Ans.} \\
 \underline{25000} \\
 200 \\
 \underline{12} \\
 2400 \\
 \underline{4} \\
 9600 \\
 \underline{9375} \\
 225
 \end{array}$$

(30)

If I buy 15 yards of cloth for 11 guineas, how many Flemish ells can I buy for 240*l.* 13*s.* 4*d.* at the same rate?

$$\begin{array}{rcl}
 \text{As } 11 \text{ } ^{\text{z.}} 11 \text{ } ^{\text{s.}} : 240 \text{ } ^{\text{z.}} 13 \text{ } ^{\text{s.}} 4 \text{ } ^{\text{d.}} :: 15 \text{ } ^{\text{yds.}} \\
 \hline
 20 \qquad \qquad \qquad 20 \qquad \qquad \qquad 4 \\
 \underline{231} \qquad \qquad \qquad \underline{4813} \qquad \qquad \qquad \underline{60 \text{ qrs.}} \\
 12 \qquad \qquad \qquad 12 \\
 \underline{2772} \qquad \qquad \qquad \underline{57760} \\
 \qquad \qquad \qquad 60 \\
 2772 \overline{) 3465600} \text{ (1250}\frac{1}{3}\frac{1}{3}\frac{1}{3} \text{ qrs.} \\
 \underline{2772} \\
 6936 \\
 \underline{5544} \\
 13920 \\
 \underline{13860} \\
 600 \\
 3 \overline{) 1250}\frac{1}{3}\frac{1}{3}\frac{1}{3} \\
 \underline{416 \text{ Fl. ells, 2 qrs. } \frac{1}{3}\frac{1}{3}\frac{1}{3} \text{ Ans.}}
 \end{array}$$

(31)

The rents of a whole parish amount to 1750*l.*, and a rate is granted of 32*l.* 16*s.* 6*d.*: what is that in the pound?

<i>£.</i>	<i>£.</i>	<i>£.</i>	<i>s.</i>	<i>d.</i>
As 1750	: 1	::	32	16 6
			20	
			656	
			12	
1750)	7878	(4½ <i>d.</i>	1½ <i>ss</i>
		7000		
		878		
		4		
		3512		
		3500		
		12		

(32)

If my horse stands me in 11½*d.* per day keeping, what will be the charge of 11 horses for the year?

<i>day.</i>	<i>days.</i>	<i>d.</i>
As 1	: 365	:: 11½
		4
		46
		365
	4)	16790
	12)	4197½
	2,0)	34,9 9½
Cost of keep. 1	}	= 17 9 9½
horse for 1 yr.	}	11
Charge of	}	= 192 <i>l.</i> 7 <i>s.</i> 8½ <i>d.</i>
11 horses	}	

(33)

A bankrupt owes in all 1490*l.* 5*s.* 10*d.*, and has in money, goods, and recoverable debts, 784*l.* 17*s.* 4*d.*: if this sum be divided among his creditors, what will they get in the pound?

<i>£.</i>	<i>s.</i>	<i>d.</i>	:	<i>£.</i>	:	:	<i>£.</i>	<i>s.</i>	<i>d.</i>
As 1490	5	10		1			784	17	4
	20			240			20		
	29805			240			15697		
	12						12		
	357670						188368		
							240		
				357670)	45208320			
							126½ <i>d.</i>	117887	
							10 <i>s.</i> 6½ <i>d.</i>	117887	

(34)

What must 40s. pay towards a tax when 652*l.* 13*s.* 4*d.* is assessed at 83*l.* 12*s.* 4*d.*?

<i>£</i>	<i>s.</i>	<i>d.</i>		<i>£</i>		<i>£</i>	<i>s.</i>	<i>d.</i>
As 652	13	4	:	40	::	83	12	4
	20			12			20	
	13053			480			1672	
	12						12	
	156640						20068	
							480	
				156640)	9632640		
							61½ <i>d.</i>	†††††
							Or 5 <i>s.</i> 1½ <i>d.</i>	†††††

(35)

Bought 3 tuns of oil for 151*l.* 14*s.*, 85 gallons of which being damaged, I desire to know how I may sell the remainder per gallon so as neither to gain nor lose by the bargain?

<i>tuns.</i>		<i>gall.</i>		<i>£</i>	<i>s.</i>
As 3	:	1	::	151	14
4				20	
12 hhds.			671	3034	(4 <i>s.</i> 6½ <i>d.</i> ⅔ Ans.
63				2684	
756 galls.				350	
sub. 85				12	
671 galls.				4200	
				4026	
				174	
				4	
				696	
				671	
				25	

(36)

What quantity of water must I add to a pipe of mountain, value 33*l.*, to reduce the first cost to 4*s.* 6*d.* per gallon?

$$\begin{array}{rclclcl}
 & \text{s.} & \text{d.} & & \text{s.} & \text{gall.} \\
 \text{As } 4 & 6 & : & 33 & : : & 1 \\
 \hline
 & 12 & & 20 & & \\
 & 54\text{d.} & & 660\text{s.} & & \\
 & & & 12 & &
 \end{array}$$

$$54 \overline{) 7920\text{d.}}$$

$$\underline{146 \text{ gall. } 2 \text{ qts. } 1\frac{1}{2} \text{ pt.}}$$

$$1 \text{ pipe} = 126$$

$$\text{Water to be added } \underline{20 \text{ gall. } 2 \text{ qts. } 1\frac{1}{2} \text{ pt.}}$$

(37)

If 15 ells of stuff, $\frac{3}{4}$ yard wide, cost 37*s.* 6*d.*, what will 40 ells of the same stuff cost, being yard wide?

$$\begin{array}{rclclcl}
 & \text{ells } \frac{3}{4} \text{ wide.} & & \text{ells } 1 \text{ qrs. wide.} & & \text{s.} & \text{d.} \\
 \text{As } 15 & : & 40 & : : & 37 & 6 \\
 \hline
 & 3 & & 4 & & 12 \\
 & 45 \text{ qrs.} & & 160 \text{ qrs.} & & 450\text{d.} \\
 & & & & & 160
 \end{array}$$

$$45 \overline{) 72000}$$

$$12 \overline{) 1600\text{d.}}$$

$$2,0 \overline{) 13,3 \text{ } 4}$$

$$\underline{6\text{l. } 13\text{s. } 4\text{d.} \text{ Ans.}}$$

(38)

Shipped for Barbadoes 500 pairs of stockings at 3*s.* 6*d.* per pair, and 1650 yards of baize at 1*s.* 3*d.* per yard; and have received in return 348 gallons of rum at 6*s.* 8*d.* per gallon, and 750 lb. of indigo at 1*s.* 4*d.* per lb.: what remains due upon my adventure?

$$\begin{array}{lcl}
 \text{As 1 pair : } 500 \text{ pairs} & : : 3 \text{ } 6 & : : 87 \text{ } 10 \text{ } 0 \text{ val. of stockings.} \\
 \text{As 1 yd. : } 1650 \text{ yds.} & : : 1 \text{ } 3 & : : 103 \text{ } 2 \text{ } 6 \text{ val. of baize.}
 \end{array}$$

$$\text{The value of goods shipped } \underline{\pounds 190 \text{ } 12 \text{ } 6}$$

$$\begin{array}{lcl}
 \text{As 1 gall. : } 348 \text{ galls.} & : : 6 \text{ } 8 & : : 116 \text{ value of rum.} \\
 \text{As 1 lb. : } 750 \text{ lbs.} & : : 1 \text{ } 4 & : : 50 \text{ value of indigo.}
 \end{array}$$

$$\text{Value of goods received } \underline{\pounds 166}$$

$$\text{Value of goods shipped} = \underline{\pounds 190 \text{ } 12 \text{ } 6}$$

$$\text{Value of goods received} = \underline{166 \text{ } 0 \text{ } 0}$$

$$\text{Remains due } \underline{24\text{l. } 12\text{s. } 6\text{d.} \text{ Ans.}}$$

(39)

What is a quarter's rent of 500 acres of land which is let for 1*l.* 15*s.* 6*d.* an acre per annum?

As 1 acre : 500 acres :: 1 ^{£.} 15 ^{s.} 6 : 887 10 yearly rent.

4) 887 10

221*l.* 17*s.* 6*d.* the quarter's rent as required.

(40)

A factor bought 19 pieces of holland cloth, which cost him 176*l.* 13*s.* at the rate of 5*s.* 3*d.* per ell Flemish : how many English ells did the 19 pieces contain?

As 5 ^{s.} 3 ^{d.} : 176 ^{£.} 13 ^{s.} :: 3 ^{gr.}

12	20
<u>63<i>d.</i></u>	<u>3533</u>
	12
	<u>42396<i>d.</i></u>
	3

63) 127188

5) 2018 qrs. 3 nls. $\frac{1}{2}$

403 E. ells, 3 qrs. 3 $\frac{1}{2}$ nls. Ans.

(41)

A person failing in trade compounds with his creditors to pay them half a guinea in the pound, and accordingly paid them 1852*l.* 13*s.* 6*d.* : what was his whole debt?

As 10 ^{s.} 6 ^{d.} : 1852 ^{£.} 13 ^{s.} 6 ^{d.} :: 1

2	20
<u>21</u>	<u>37053</u>
	2

21) 74107

3528*l.* 18*s.* 1*d.* $\frac{1}{2}$

(42)

If an ounce of gold cost 5 guineas, what is the value of 1 grain?

As 1 ^{oz.} :	1 ^{gr.} ::	105 ^{s.}
480		<u>12</u>
<u>480</u> grs.	480)	1260 <i>d.</i>

2 $\frac{1}{2}$ *d.* $\frac{1}{2}$ q.

(43)

If 3 cwt. of tea cost 40*l.* 13*s.*, at how much must it be sold per pound to gain 10*l.* by the whole?

As 3 ^{cwt.} :	1 ^{lb.} ::	50 ^{£.} 13
112		<u>20</u>

336lb. 336) 1013

3*s.* 0*d.* $\frac{1}{2}$ f.

(44)

How many pieces of holland, each containing 15 ells Flemish, may be bought for 30*l.* 16*s.* 5*d.* at the rate of 5*s.* 3*d.* per ell English?

s. d.	:	s. s. d.	::	qrs.
As 5 3	:	30 16 5	::	5
12		20		
<u>63<i>d.</i></u>		<u>616</u>		
		12		
15 Fl. ells		<u>7397<i>d.</i></u>		
3		5		
<u>45 qrs.</u>	63	<u>36985</u>		
	45	<u>587<i>½</i> qrs.</u>		
		<u>13<i>⅔</i> pieces required.</u>		

(45)

If a gentleman's estate be worth 384*l.* 16*s.* a year, and the land-tax be assessed at 2*s.* 9*½d.* in the pound, what is his net annual income?

s.	:	s. s.	::	s. d.
As 1	:	384 16	::	2 9 <i>½</i>
20		20		12
<u>20<i>s.</i></u>		<u>7696<i>s.</i></u>		<u>33<i>d.</i></u>
		134		4
		30784		<u>134<i>f.</i></u>
		100048		
20)	1031264		
4)	51563 <i>½</i>		
12)	12890 <i>½</i>		
2,0)	107,4 2 <i>½</i>		
		£53 14 2 <i>½</i> $\frac{1}{2}$ the tax on 384 <i>l.</i> 16 <i>s.</i>		
		384 16		
		<u>£331 1 9<i>½</i> net annual income required.</u>		

(46)

The circumference of the earth is about 25000 miles; at what rate per hour must a body be carried to pass completely round it in 23 hours 56 minutes, the length of a sidereal day?

<i>hr. min.</i>	<i>hr.</i>	<i>miles.</i>
As 23 56 :	1 :	25000
60	60	
<u>1436</u>	<u>60</u>	
	25000	
1436)	1500000	
	1044 $\frac{1}{4}$	miles
	[per hour. <i>Ans.</i>	

(48)

If 100 workmen can finish a piece of work in 12 days, how many are sufficient to do the same in 3 days?

<i>days.</i>	<i>days.</i>	<i>workmen.</i>
As 3 :	12 :	100
	12	
3)	1200	
	400	men.

(50)

How many yards of matting 2 feet 6 inches broad will cover a floor that is 27 feet long and 20 feet broad?

<i>ft. in.</i>	<i>ft.</i>	<i>ft.</i>
As 2 6 :	20 :	27
12	12	
<u>30</u>	<u>240</u>	
	27	
3,0)	648,0	
3)	216	feet long.
	72	yards. <i>Ans.</i>

(47)

What quantity of shalloon that is 3 quarters wide will line $7\frac{1}{2}$ yards of cloth that is $1\frac{1}{2}$ yard wide?

<i>qr.</i>	<i>yd. qr.</i>	<i>yd. qr.</i>
As 3 :	1 2 :	7 2
	$\frac{4}{6}$	$\frac{4}{30}$
	6	qrs.
	3)	180
	4)	60 qrs.
		15 yds.

(49)

How much in length that is $4\frac{1}{2}$ inches broad will make a square foot?

<i>in.</i>	<i>in.</i>	<i>in.</i>
As $4\frac{1}{2}$:	12 :	12
Or 9 half in. :	24 :	12
	24	
	9)	288
		32 in.

(51)

How many yards of cloth 3 qrs. wide are equal in measure to 30 yards 5 qrs. wide?

<i>qr.</i>	<i>qr.</i>	<i>yd.</i>
As 3 :	5 :	30
	5	
	3)	150
		50 yds. <i>Ans.</i>

(52)

If, when the price of a bushel of wheat is 6s. 3d., the penny loaf weighs 9 ounces, what ought it to weigh when wheat is at 8s. 2½d. per bushel?

$$\begin{array}{r}
 \text{As } 8 \text{ } 2\frac{1}{2} \text{ } : \text{ } 6 \text{ } 3 \text{ } : : \text{ } 9 \\
 \hline
 12 \quad 12 \\
 98 \quad 75 \\
 4 \quad 4 \\
 \hline
 394 \quad 300 \\
 \hline
 9
 \end{array}$$

394) 2700 (6 oz. 13 dr. ⅓

$$\begin{array}{r}
 2364 \\
 \hline
 336 \\
 16
 \end{array}$$

$$\begin{array}{r}
 394) 5376 \\
 \hline
 394 \\
 \hline
 1436 \\
 \hline
 1182 \\
 \hline
 254
 \end{array}$$

(53)

How many yards of stuff 3 qrs. broad will line a cloak that is 5½ yards in length and 1½ yard broad?

$$\begin{array}{r}
 \text{qr. yd. yd.} \\
 \text{As } 3 : 1\frac{1}{2} : : 5\frac{1}{2} \\
 \hline
 4 \quad 4 \\
 5 \text{ qrs. } 22 \text{ qrs.} \\
 \hline
 22
 \end{array}$$

$$3) 110$$

$$4) 36 \text{ qrs. } 2\frac{1}{2} \text{ nls.}$$

$$9 \text{ yds. } 0 \text{ qr. } 2\frac{1}{2} \text{ nls.}$$

(54)

If 4½ cwt. may be carried 36 miles for 35s., how many pounds can I have carried 20 miles for the same money?

$$\begin{array}{r}
 \text{mi. mi. cwt.} \\
 \text{As } 20 : 36 : : 4\frac{1}{2} \\
 \hline
 4 \\
 18 \text{ qrs.} \\
 \hline
 28
 \end{array}$$

$$504 \text{ lbs.}$$

$$36$$

$$3024$$

$$1512$$

$$2,0) 1814,4$$

$$907 \text{ lbs. } 3 \text{ oz. } 3 \text{ dr. } \frac{4}{5}$$

(55)

How much in length that is 13½ poles in breadth must be taken to contain an acre?

$$\begin{array}{r}
 \text{po. po. po.} \\
 \text{As } 13\frac{1}{2} : 4 : : 40 \\
 \hline
 2 \quad 2 \quad 8 \\
 27 \quad 8 \quad 27) 320
 \end{array}$$

$$11 \text{ po. } 4 \text{ yds. } 2 \text{ ft. } \frac{1}{2} \text{ in. Ans.}$$

(56)

How many yards of canvas that is ell wide will line 20 yards of say that is 3 qrs. wide?

$$\begin{array}{rcl}
 \text{gr.} & \text{gr.} & \text{yd.} \\
 \text{As } 5 & : 3 :: 20 \\
 & 20 \\
 5 & \overline{) 60} \\
 & \underline{12} \text{ yds. } \text{Ans.}
 \end{array}$$

(57)

If 30 men can perform a piece of work in 11 days, how many men will accomplish another piece of work, four times as large, in a fifth part of the time?

$$\begin{array}{rcl}
 \text{piece. pieces. men.} \\
 \text{As } 1 & : 4 :: 30 \\
 & 4 \\
 & \underline{120} \text{ men.}
 \end{array}$$

$$\begin{array}{rcl}
 \text{day. days. men.} \\
 \text{As } 1 & : 5 :: 120 \\
 & 5 \\
 \text{Ans. } & \underline{600} \text{ men.}
 \end{array}$$

(58)

A wall that is to be built to the height of 27 feet was raised 9 feet by 12 men in 6 days: how many men must be employed to finish the wall in 4 days at the same rate of working?

Since 9 feet are finished, there are 18 feet to be built.

$$\begin{array}{rcl}
 \text{ft.} & \text{ft.} & \text{men.} \\
 \text{As } 9 & : 18 :: 12 \\
 & 12 \\
 9 & \overline{) 216} \\
 & \underline{24} \text{ men.}
 \end{array}$$

Now, if 24 men could finish the wall in 6 days, how many could finish it in 4 days?

$$\begin{array}{rcl}
 \text{days. days. men.} \\
 \text{As } 4 & : 6 :: 24 \\
 & 24 \\
 4 & \overline{) 144} \\
 & \underline{36} \text{ men. } \text{Ans.}
 \end{array}$$

(59)

How many yards of paper, $1\frac{1}{2}$ yard wide, will be sufficient to hang a room which is 20 yards in circumference and 4 in height?

$$\begin{array}{r}
 \text{yd.} \quad \text{yd.} \quad \text{yd.} \\
 \text{As } 1\frac{1}{2} : 4 :: 20 \\
 \underline{4} \quad \underline{4} \\
 5 \text{ qrs. } 16 \text{ qrs.} \\
 \underline{\quad} \quad \underline{20} \\
 5 \overline{) 320} \\
 \underline{64 \text{ yds.}} \text{ Ans.}
 \end{array}$$

(61)

How many Venetian ducats, at 4s. 4d. each, are equal to 730 rix-dollars, at 4s. $5\frac{1}{2}$ d. each?

$$\begin{array}{r}
 \text{s. d.} \quad \text{s. d.} \quad \text{rix-dol.} \\
 \text{As } 4 \ 4 : 4 \ 5\frac{1}{2} :: 730 \\
 \underline{12} \quad \underline{12} \\
 \underline{52} \quad \underline{53} \\
 \underline{4} \quad \underline{4} \\
 \underline{208 \text{ f.}} \quad \underline{215 \text{ f.}} \\
 \underline{\quad} \quad \underline{730} \\
 \underline{\quad} \quad \underline{6450} \\
 \underline{\quad} \quad \underline{1505} \\
 208 \overline{) 156950} \\
 \underline{754\frac{5}{8}} \text{ Ans.}
 \end{array}$$

(60)

If 14 cwt. be carried 136 miles for 5l. 5s., how many cwt. may be carried 79 $\frac{1}{2}$ miles for the same money?

$$\begin{array}{r}
 \text{miles.} \quad \text{miles.} \quad \text{cwt.} \\
 \text{As } 79\frac{1}{2} : 136 :: 14 \\
 \underline{3} \quad \underline{3} \\
 \underline{238} \quad \underline{408} \\
 \underline{\quad} \quad \underline{14} \\
 238 \overline{) 5712} \\
 \underline{\quad} \quad \underline{24 \text{ cwt.}} \text{ Ans.}
 \end{array}$$

(62)

How many yards of canvas, which is 1 English ell wide, will line 15 French ells of cloth, which is 1 Flemish ell wide?

$$\begin{array}{r}
 \text{qrs.} \quad \text{Fl. ell.} \quad \text{Fr. ells.} \\
 \text{As } 5 : 1 :: 15 \\
 \underline{3} \quad \underline{6} \\
 \underline{3 \text{ qrs.}} \quad \underline{90 \text{ qrs.}} \\
 \underline{90} \\
 5 \overline{) 270} \\
 4 \overline{) 54 \text{ qrs.}} \\
 \underline{13 \text{ yds. } 2 \text{ qrs.}}
 \end{array}$$

(63)

If a person drink 20 pints of wine per month when it costs 8s. a gallon, how many pints may he drink in the same time, without increasing the expense, when wine costs 10s. per gallon?

$$\begin{array}{r}
 \text{s.} \quad \text{s.} \quad \text{pints.} \\
 \text{As } 10 : 8 :: 20 \\
 \underline{8} \\
 10 \overline{) 160} \\
 \underline{16 \text{ pints.}} \text{ Ans.}
 \end{array}$$

(64)

If a tailor can make a coat and waistcoat with 3 yds. and 3 qrs. of cloth, which is one yard and a half in breadth, how many yards will he require to make the same when the breadth is only 2 qrs. 2 nls.?

$$\begin{array}{rclcl}
 \text{qr.} & \text{nl.} & & \text{yd.} & & \text{yd.} & \text{qr.} \\
 \text{As } 2 & 2 & : & 1\frac{1}{2} & :: & 3 & 3 \\
 \hline
 & 4 & & 4 & & 4 & \\
 10 & \text{nls.} & & 6 & \text{qrs.} & 15 & \text{qrs.} \\
 \hline
 & & & 4 & & 24 & \\
 24 & \text{nls.} & 10 & \overline{) 360} & & & \\
 & & & 4 & \overline{) 36} & \text{qrs.} & \\
 & & & & 9 & \text{yds.} & \text{Ans.}
 \end{array}$$

NOTE.—It has been thought proper to give the questions and their solutions at full length in the Rule of Three, that the reader may see at one view both the conditions and the solutions of the questions.

COMPOUND PROPORTION, p. 58.

$$(2) \quad \text{As } \left\{ \begin{array}{l} 16 \text{ acres} \\ 7 \text{ days} \end{array} \right\} : \left\{ \begin{array}{l} 24 \text{ acres} \\ 20 \text{ days} \end{array} \right\} :: 7 \text{ horses.}$$

$$\text{Ans.} = \frac{24 \times 20 \times 7}{16 \times 7} = 30 \text{ horses.}$$

$$(3) \quad \text{As } \left\{ \begin{array}{l} 9 \text{ people} \\ 8 \text{ months} \end{array} \right\} : \left\{ \begin{array}{l} 24 \text{ people} \\ 16 \text{ months} \end{array} \right\} :: 120\text{l.}$$

$$\text{Ans.} = \frac{24 \times 16 \times 120}{9 \times 8} = 640\text{l.}$$

$$(4) \quad \text{As } \left\{ \begin{array}{l} 24 \text{ yards} \\ 3 \text{ days} \end{array} \right\} : \left\{ \begin{array}{l} 18 \text{ yards} \\ 6 \text{ days} \end{array} \right\} :: 8 \text{ men.}$$

$$\text{Ans.} = \frac{18 \times 6 \times 8}{24 \times 3} = 12 \text{ men.}$$

* The number of horses required must evidently depend, not on the number of acres only, or upon the number of days only, but upon these two quantities jointly. Hence $16 \times 7 : 24 \times 20 :: 7 : \text{horses required.}$

$$(5) \quad \text{As } \left\{ \begin{array}{l} 2 \text{ men} \\ 6\frac{1}{2} \text{ days} \end{array} \right\} : \left\{ \begin{array}{l} 18 \text{ men} \\ 14 \text{ days} \end{array} \right\} :: 12\frac{1}{2} \text{ rods.}$$

$$\text{Ans.} = \frac{18 \times 14 \times 12\frac{1}{2}}{2 \times 6\frac{1}{2}} = 247\frac{3}{4} \text{ rods.}$$

$$(6) \quad \text{As } \left\{ \begin{array}{l} 351 \text{ quarters} \\ 5 \text{ months} \end{array} \right\} : \left\{ \begin{array}{l} 1464 \text{ quarters} \\ 7 \text{ months} \end{array} \right\} :: 939 \text{ men.}$$

$$\text{Ans.} = \frac{1464 \times 7 \times 939}{351 \times 5} = 5483\frac{2}{3} \text{ men.}$$

$$(7) \quad \text{As } \left\{ \begin{array}{l} 5 \text{ cwt. 3 qrs.} \\ 150 \text{ miles} \end{array} \right\} : \left\{ \begin{array}{l} 7 \text{ cwt. 2 qrs. 25 lb.} \\ 64 \text{ miles} \end{array} \right\} :: 3\text{l. } 7\text{s. } 4\text{d.}$$

or $\left\{ \begin{array}{l} 644 \text{ lbs.} \\ 150 \text{ miles} \end{array} \right\} : \left\{ \begin{array}{l} 865 \text{ lbs.} \\ 64 \text{ miles} \end{array} \right\} :: 808\text{d.}$

$$\text{Ans.} = \frac{865 \times 64 \times 808}{644 \times 150} \text{ pence} = 1\text{l. } 18\text{s. } 7\text{d. } \frac{1}{4}\frac{1}{4}\frac{1}{5}$$

$$(8) \quad \text{As } \left\{ \begin{array}{l} 24 \text{ men} \\ 9 \text{ hours} \\ 230 \text{ yards} \\ 3 \text{ yds. wide} \\ 2 \text{ yds. deep} \end{array} \right\} : \left\{ \begin{array}{l} 248 \text{ men} \\ 11 \text{ hours} \\ 420 \text{ yards} \\ 5 \text{ yds. wide} \\ 3 \text{ yds. deep} \end{array} \right\} :: 5 \text{ days.}$$

$$\text{Ans.} = \frac{248 \times 11 \times 420 \times 5 \times 3 \times 5}{24 \times 9 \times 230 \times 3 \times 2} = 288 \text{ days } \frac{5}{7}.$$

$$(9) \quad \text{As } \left\{ \begin{array}{l} 300 \text{ miles} \\ 16 \text{ hours} \end{array} \right\} : \left\{ \begin{array}{l} 600 \text{ miles} \\ 12 \text{ hours} \end{array} \right\} :: 10 \text{ days.}$$

$$\text{Ans.} = \frac{600 \times 12 \times 10}{300 \times 16} = 15 \text{ days.}$$

$$(10) \quad \text{As } \left\{ \begin{array}{l} 7 \text{ persons} \\ 12 \text{ days} \end{array} \right\} : \left\{ \begin{array}{l} 14 \text{ persons} \\ 365 \text{ days} \end{array} \right\} :: 1 \text{ bar.}$$

$$\text{Ans.} = \frac{14 \times 365 \times 1}{7 \times 12} = 60\frac{1}{2} \text{ barrels.}$$

$$(11) \quad \text{As } \left\{ \begin{array}{l} 6 \text{ workmen} \\ 21 \text{ weeks} \end{array} \right\} : \left\{ \begin{array}{l} 14 \text{ workmen} \\ 46 \text{ weeks} \end{array} \right\} :: 120\text{l.}$$

$$\text{Ans.} = \frac{14 \times 46 \times 120}{6 \times 21} = 613\text{l. } 6\text{s. } 8\text{d.}$$

$$(12) \quad \text{As } \left\{ \begin{array}{l} 5 \text{ feet} \\ 2\frac{1}{2} \text{ in. broad} \\ 1\frac{1}{2} \text{ in. thick} \end{array} \right\} : \left\{ \begin{array}{l} 7 \text{ feet} \\ 4 \text{ in. broad} \\ 3 \text{ in. thick} \end{array} \right\} :: 49 \text{ lbs.}$$

$$\text{Ans.} = \frac{7 \times 4 \times 3 \times 49}{5 \times 2\frac{1}{2} \times 1\frac{1}{2}} = 188\frac{4}{5} \text{ lbs.}$$

$$(13) \text{ As } \left\{ \begin{array}{l} 26 \text{ men} \\ 13 \text{ hours} \\ 176 \text{ yards} \\ 3 \text{ yds. wide} \\ 2 \text{ yds. deep} \end{array} \right\} : \left\{ \begin{array}{l} 138 \text{ men} \\ 10 \text{ hours} \\ 420 \text{ yards} \\ 4 \text{ yds. wide} \\ 3 \text{ yds. deep} \end{array} \right\} :: 6 \text{ days}$$

$$\text{Ans.} = \frac{138 \times 10 \times 420 \times 4 \times 3 \times 6}{26 \times 13 \times 176 \times 3 \times 2} = 116 \text{ days, 22 hours } 14\frac{2}{3}.$$

PRACTICE, p. 61.

CASE I. *When the price is less than a penny.*

$$\begin{array}{r} 3456 \text{ at } \frac{1}{4}d. \\ \frac{1}{4} \text{ is } \frac{1}{4}) 3456 \\ 12 \overline{) 864d.} \\ 2,0 \overline{) 7,2s.} \\ \hline 3l. 12s. \text{ Ans.} \end{array}$$

$$\begin{array}{r} 846 \text{ at } \frac{1}{4}d. \\ \frac{1}{4} \text{ is } \frac{1}{4}) 423 \\ \frac{1}{4} \text{ is } \frac{1}{4}) 211 \frac{1}{4} \\ 12 \overline{) 634 \frac{1}{4}} \\ 2,0 \overline{) 5,2 \text{ } 10\frac{1}{4}} \\ \hline 2l. 12s. 10\frac{1}{4}d. \text{ Ans.} \end{array}$$

$$\begin{array}{r} 347 \text{ at } \frac{1}{4}d. \\ \frac{1}{4} \text{ is } \frac{1}{4}) 347 \\ 12 \overline{) 173\frac{1}{2}d.} \\ \hline 14s. 5\frac{1}{2}d. \text{ Ans.} \end{array}$$

$$\begin{array}{r} 810 \text{ at } \frac{1}{4}d. \\ \frac{1}{4} \text{ is } \frac{1}{4}) 405 \\ \frac{1}{4} \text{ is } \frac{1}{4}) 202 \frac{1}{4} \\ 12 \overline{) 607 \frac{1}{4}} \\ 2,0 \overline{) 5,0 \text{ } 7\frac{1}{4}} \\ \hline 2l. 10s. 7\frac{1}{4}d. \text{ Ans.} \end{array}$$

CASE II. *When the price is an aliquot part of a shilling.*

$$\begin{array}{r} 437 \text{ at } 1d. \\ 1d. \text{ is } \frac{1}{12}s.) 437 \\ 2,0 \overline{) 3,6s. 5d.} \\ \hline 1l. 16s. 5d. \text{ Ans.} \end{array}$$

$$\begin{array}{r} 5275 \text{ at } 2d. \\ 2d. \text{ is } \frac{1}{6}s.) 5275 \\ 2,0 \overline{) 87,9s. 2d.} \\ \hline 43l. 19s. 2d. \text{ Ans.} \end{array}$$

$$\begin{array}{r} 352 \text{ at } 1\frac{1}{2}d. \\ 1\frac{1}{2}d. \text{ is } \frac{1}{8}s.) 352 \\ 2,0 \overline{) 4,4s.} \\ \hline 2l. 4s. \text{ Ans.} \end{array}$$

$$\begin{array}{r} 1776 \text{ at } 3d. \\ 3d. \text{ is } \frac{1}{4}s.) 1776 \\ 2,0 \overline{) 44,4s.} \\ \hline 22l. 4s. \text{ Ans.} \end{array}$$

$$\begin{array}{r}
 6771 \text{ at } 4d. \\
 4d. \text{ is } \frac{1}{4}s.) \overline{6771} \\
 \quad 2,0) \overline{225,7s.} \\
 \quad \underline{112l. \ 17s.} \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 889 \text{ at } 6d. \\
 6d. \text{ is } \frac{1}{2}s.) \overline{889} \\
 \quad 2,0) \overline{44,9s. \ 6d.} \\
 \quad \underline{22l. \ 9s. \ 6d.} \text{ Ans.}
 \end{array}$$

CASE III.

$$\begin{array}{r}
 372 \text{ at } 1\frac{1}{2}d. \\
 1\frac{1}{2} \text{ is } \frac{1}{3} \mid \overline{46 \ 6} \\
 \frac{1}{3} \text{ is } \frac{1}{3} \mid \overline{7 \ 9} \\
 \quad 2,0) \overline{5,4 \ 3} \\
 \quad \underline{2l. \ 14s. \ 3d.} \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 2700 \text{ at } 7\frac{1}{2}d. \\
 6d. \text{ is } \frac{1}{2} \mid \overline{1350} \\
 1d. \text{ is } \frac{1}{2} \mid \overline{225} \\
 \frac{1}{2} \text{ is } \frac{1}{2} \mid \overline{56 \ 3} \\
 \quad 2,0) \overline{163,1 \ 3} \\
 \quad \underline{81l. \ 11s. \ 3d.} \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 325 \text{ at } 2\frac{1}{2}d. \\
 2d. \text{ is } \frac{1}{2} \mid \overline{54 \ 2} \\
 \frac{1}{2} \text{ is } \frac{1}{2} \mid \overline{13 \ 6\frac{1}{2}} \\
 \quad 2,0) \overline{6,7 \ 8\frac{1}{2}} \\
 \quad \underline{3l. \ 7s. \ 8\frac{1}{2}d.} \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 2150 \text{ at } 9\frac{1}{2}d. \\
 6d. \text{ is } \frac{1}{2} \mid \overline{1075} \\
 3d. \text{ is } \frac{1}{2} \mid \overline{537 \ 6} \\
 \frac{1}{2} \text{ is } \frac{1}{2} \mid \overline{134 \ 4\frac{1}{2}} \\
 \quad 2,0) \overline{174,6 \ 10\frac{1}{2}} \\
 \quad \underline{87l. \ 6s. \ 10\frac{1}{2}d.} \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 827 \text{ at } 4\frac{1}{2}d. \\
 4d. \text{ is } \frac{1}{2} \mid \overline{275 \ 8} \\
 \frac{1}{2} \text{ is } \frac{1}{2} \mid \overline{34 \ 5\frac{1}{2}} \\
 \quad 2,0) \overline{31,0 \ 1\frac{1}{2}} \\
 \quad \underline{15l. \ 10s. \ 1\frac{1}{2}d.} \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 1720 \text{ at } 11\frac{1}{2}d. \\
 6d. \text{ is } \frac{1}{2} \mid \overline{860} \\
 3d. \text{ is } \frac{1}{2} \mid \overline{430} \\
 2d. \text{ is } \frac{1}{2} \text{ of } 6d. \mid \overline{286 \ 8} \\
 \frac{1}{2} \text{ is } \frac{1}{2} \mid \overline{71 \ 8} \\
 \quad 2,0) \overline{164,8 \ 4} \\
 \quad \underline{82l. \ 8s. \ 4d.} \text{ Ans.}
 \end{array}$$

CASE IV.

$$\begin{array}{r}
 2757 \text{ at } 1s. \\
 \quad 1 \\
 2,0) \overline{275,7} \\
 \quad \underline{137l. \ 17s.} \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 3271 \text{ at } 5s. \\
 \quad 2 \\
 \quad \underline{654 \ 4} \\
 \quad \overline{163 \ 11} \\
 \quad \underline{817l. \ 15s.} \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 2643 \text{ at } 2s. \\
 \quad 1 \\
 \quad \underline{264l. \ 6s.} \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 872 \text{ at } 8s. \\
 \quad 4 \\
 \quad \underline{348l. \ 16s.} \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 372 \text{ at } 11s. \\
 \underline{5} \\
 186 \\
 \frac{1}{10}) 18 \ 12 \\
 \underline{204l. \ 12s. \ Ans.}
 \end{array}$$

$$\begin{array}{r}
 5271 \text{ at } 14s. \\
 \underline{7} \\
 3689l. \ 14s. \ Ans.
 \end{array}$$

$$\begin{array}{r}
 3142 \text{ at } 17s. \\
 \underline{8} \\
 2513 \ 12 \\
 \frac{1}{10}) 157 \ 2 \\
 \underline{2670l. \ 14s. \ Ans.}
 \end{array}$$

$$\begin{array}{r}
 264 \text{ at } 19s. \\
 \underline{9} \\
 237 \ 12 \\
 \frac{1}{10}) 13 \ 4 \\
 \underline{250l. \ 16s. \ Ans.}
 \end{array}$$

CASE V.

$$\begin{array}{r}
 7150 \text{ at } 1s. \ 8d. \\
 1s. \ 8d. \text{ is } \frac{1}{8}) 7150 \\
 \underline{Ans. \ 595l. \ 16s. \ 8d.}
 \end{array}$$

$$\begin{array}{r}
 2715 \text{ at } 2s. \ 6d. \\
 2s. \ 6d. \text{ is } \frac{1}{4}) 2715 \\
 \underline{Ans. \ 339l. \ 7s. \ 6d.}
 \end{array}$$

$$\begin{array}{r}
 3150 \text{ at } 3s. \ 4d. \\
 3s. \ 4d. \text{ is } \frac{1}{3}) 3150 \\
 \underline{Ans. \ 525l.}
 \end{array}$$

$$\begin{array}{r}
 2710 \text{ at } 6s. \ 8d. \\
 6s. \ 8d. \text{ is } \frac{1}{2}) 2710 \\
 \underline{Ans. \ 903l. \ 6s. \ 8d.}
 \end{array}$$

When the price is no aliquot part of a pound.

EXAMPLES.

$$\begin{array}{r}
 7211 \text{ at } 1s. \ 3d. \\
 1s. \text{ is } \frac{1}{4}) 360 \ 11 \\
 3d. \text{ is } \frac{1}{4}) 90 \ 2 \ 9 \\
 \underline{\underline{£450 \ 11 \ 9 \ Ans.}}
 \end{array}$$

$$\begin{array}{r}
 2701 \text{ at } 3s. \ 2d. \\
 2s. \text{ is } \frac{1}{2}) 270 \ 2 \\
 1s. \text{ is } \frac{1}{4}) 135 \ 1 \\
 2d. \text{ is } \frac{1}{4}) 22 \ 10 \ 2 \\
 \underline{\underline{£427 \ 13 \ 2 \ Ans.}}
 \end{array}$$

$$\begin{array}{r}
 801 \text{ at } 10s. \ 9d. \\
 10s. \text{ is } \frac{1}{2}) 400 \ 10 \\
 6d. \text{ is } \frac{1}{4}) 20 \ 0 \ 6 \\
 3d. \text{ is } \frac{1}{4}) 10 \ 0 \ 3 \\
 \underline{\underline{£430 \ 10 \ 9 \ Ans.}}
 \end{array}$$

$$\begin{array}{r}
 841 \text{ at } 13s. \ 2d. \\
 10s. \text{ is } \frac{1}{2}) 420 \ 10 \\
 2s. \text{ is } \frac{1}{4}) 84 \ 2 \\
 1s. \text{ is } \frac{1}{4}) 42 \ 1 \\
 2d. \text{ is } \frac{1}{4}) 7 \ 0 \ 2 \\
 \underline{\underline{£553 \ 13 \ 2 \ Ans.}}
 \end{array}$$

$$\begin{array}{r}
 2547 \text{ at } 7s. \ 3d. \\
 5s. \text{ is } \frac{1}{4} \quad \overline{636 \ 15} \\
 2s. \text{ is } \frac{1}{2} \quad \overline{254 \ 14} \\
 3d. \text{ is } \frac{1}{4} \quad \overline{31 \ 16 \ 9} \\
 \hline
 \pounds 923 \ 5 \ 9 \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 309 \text{ at } 17s. \ 3d. \\
 10s. \text{ is } \frac{1}{2} \quad \overline{154 \ 10} \\
 5s. \text{ is } \frac{1}{2} \quad \overline{77 \ 5} \\
 1s. \text{ is } \frac{1}{4} \quad \overline{15 \ 9} \\
 1s. \ 3d. \text{ is } \frac{1}{4} \text{ of } 5s. \quad \overline{19 \ 6 \ 3} \\
 \hline
 \pounds 266 \ 10 \ 3
 \end{array}$$

$$\begin{array}{r}
 807 \text{ at } 16s. \ 5d. \\
 10s. \text{ is } \frac{1}{2} \quad \overline{403 \ 10} \\
 5s. \text{ is } \frac{1}{2} \quad \overline{201 \ 15} \\
 1s. \ 3d. \text{ is } \frac{1}{4} \quad \overline{50 \ 8 \ 9} \\
 2d. \text{ is } \frac{1}{8} \text{ of } 5s. \quad \overline{6 \ 14 \ 6} \\
 \hline
 \pounds 662 \ 8 \ 3
 \end{array}$$

$$\begin{array}{r}
 846 \text{ at } 18s. \ 6d. \\
 10s. \text{ is } \frac{1}{2} \quad \overline{423} \\
 5s. \text{ is } \frac{1}{2} \quad \overline{211 \ 10} \\
 2s. \ 6d. \text{ is } \frac{1}{2} \quad \overline{105 \ 15} \\
 1s. \text{ is } \frac{1}{4} \text{ of } 5s. \quad \overline{42 \ 6} \\
 \hline
 \pounds 782 \ 11 \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 969 \text{ at } 19s. \ 11d. \\
 10s. \text{ is } \frac{1}{2} \quad \overline{484 \ 10} \\
 6s. \ 8d. \text{ is } \frac{1}{2} \quad \overline{323 \ 0} \\
 2s. \ 6d. \text{ is } \frac{1}{2} \text{ of } 10s. \quad \overline{121 \ 2 \ 6} \\
 6d. \text{ is } \frac{1}{2} \quad \overline{24 \ 4 \ 6} \\
 3d. \text{ is } \frac{1}{2} \quad \overline{12 \ 2 \ 3} \\
 \hline
 \pounds 964 \ 19 \ 3 \text{ Ans.}
 \end{array}$$

CASE VI.

$$\begin{array}{r}
 875 \text{ at } 1s. \ 4\frac{1}{2}d. \\
 1s. \text{ is } \frac{1}{4} \quad \overline{43 \ 15} \\
 4d. \text{ is } \frac{1}{2} \quad \overline{14 \ 11 \ 8} \\
 \frac{1}{2} \text{ is } \frac{1}{4} \quad \overline{1 \ 16 \ 5\frac{1}{2}} \\
 \frac{1}{4} \text{ is } \frac{1}{2} \quad \overline{0 \ 18 \ 2\frac{1}{2}} \\
 \hline
 \pounds 61 \ 1 \ 4\frac{1}{2} \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 7524 \text{ at } 3s. \ 5\frac{1}{2}d. \\
 2s. \ 6d. \text{ is } \frac{1}{2} \quad \overline{940 \ 10} \\
 10d. \text{ is } \frac{1}{2} \quad \overline{313 \ 10} \\
 1d. \text{ is } \frac{1}{4} \quad \overline{31 \ 7} \\
 \frac{1}{2}d. \text{ is } \frac{1}{2} \quad \overline{15 \ 13 \ 6} \\
 \hline
 \pounds 1301 \ 0 \ 6 \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 2572 \text{ at } 13s. \ 7\frac{1}{2}d. \\
 10s. \text{ is } \frac{1}{2} \quad \overline{1286} \\
 2s. \ 6d. \text{ is } \frac{1}{2} \quad \overline{321 \ 10} \\
 1s. \text{ is } \frac{1}{4} \quad \overline{128 \ 12} \\
 1\frac{1}{2}d. \text{ is } \frac{1}{2} \quad \overline{16 \ 1 \ 6} \\
 \hline
 \pounds 1752 \ 3 \ 6 \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 1603 \text{ at } 16s. \ 10\frac{1}{2}d. \\
 10s. \text{ is } \frac{1}{2} \quad \overline{801 \ 10} \\
 4s. \text{ is } \frac{1}{2} \quad \overline{320 \ 12} \\
 2s. \text{ is } \frac{1}{2} \quad \overline{160 \ 6} \\
 6d. \text{ is } \frac{1}{2} \quad \overline{40 \ 1 \ 6} \\
 3d. \text{ is } \frac{1}{2} \quad \overline{20 \ 0 \ 9} \\
 1\frac{1}{2}d. \text{ is } \frac{1}{2} \quad \overline{10 \ 0 \ 4\frac{1}{2}} \\
 \hline
 \pounds 1352 \ 10 \ 7\frac{1}{2} \text{ Ans.}
 \end{array}$$

3715 at 9s. 4½d.		2710 at 19s. 2½d.	
5s. is ½	928 15	10s. is ½	1355
4s. is ½	743 0	5s. is ½	677 10
4d. is ⅙	61 18 4	3s. 4d. is ½	451 13 4
½ is ½	7 14 9½	10d. is ½	112 18 4
	<u>£1741 8 1½ Ans.</u>	½d. is ⅙	5 12 11
			<u>£2602 14 7 Ans.</u>

430 at 19s. 6½d.	
10s. is ½	215
5s. is ½	107 10
4s. is ½	86 0
6d. is ½	10 15
½ is ⅙	0 8 11½
	<u>£419 13 11½ Ans.</u>

CASE VII.

137 at 1l. 17s. 6½d.	
	1
	<u>137</u>
10s. is ½	68 10
5s. is ½	34 5
2s. 6d. is ½	17 2 6
½d. is ⅙	0 2 10½
	<u>£257 0 4½ Ans.</u>

937 at 4l. 15s. 10½d.	
	4
	<u>3788</u>
10s. is ½	473 10
5s. is ½	236 15
6d. is ⅙	23 13 6
3d. is ⅓	11 16 9
1d. is ⅓	3 18 11
½d. is ⅓	0 19 8½
	<u>£4538 13 10½ Ans.</u>

457 at 14l. 17s. 9½d.

14

6398

10s. is ½	228	10
5s. is ½	114	5
2s. 6d. is ½	57	2 6
3d. is ⅓	5	14 3
½ is ¼	0	19 0½

£6804 10 9½ Ans.

713 at 19l. 19s. 11½d.

19

13547

10s. is ½	356	10
5s. is ½	178	5
3s. 4d. is ½	118	16 8
1s. is ¼ of 5s.	35	13 0
6d. is ½	17	16 6
1½ is ¾	4	9 1½
¼ is ⅛	0	14 10½

£14259 5 1½ Ans.

CASE VIII.

273½ at 2s. 6d.

2s. 6d. is ½	34	2 6
for ½	0	7½
£34	3	1½

937½ at 3l. 17s. 8d.

3

2811

10s. is ½	468	10
5s. is ½	234	5
2s. 6d. is ½	117	2 6
2d. is ⅓	7	16 2
for the ½	1	18 10
£3640	12	6 Ans.

139½ at 1l. 19s. 4d.

1

139

10s. is ½	69	10
5s. is ½	34	15
4s. is ½	27	16
4d. is ⅓	2	6 4
for ½	19	8
for ¼	9	10

£274 16 10 Ans.

284½ at 2l. 10s. 6d.

2

568

10s. is ½	142	-
6d. is ⅓	7	2
for ½	1	5 3

£718 7 3 Ans.

$$\begin{array}{r}
 371\frac{1}{2} \text{ at } 4\text{l. } 13\text{s. } 7\text{d.} \\
 \underline{4} \\
 1484 \\
 \begin{array}{l}
 10\text{s. is } \frac{1}{4} \quad | \quad 185 \quad 10 \\
 3\text{s. } 4\text{d. is } \frac{1}{4} \quad | \quad 61 \quad 16 \quad 8 \\
 2\text{d. is } \frac{1}{10} \quad | \quad 3 \quad 1 \quad 10 \\
 1\text{d. is } \frac{1}{4} \quad | \quad 1 \quad 10 \quad 11 \\
 \text{for } \frac{1}{4} \quad | \quad 2 \quad 6 \quad 9\frac{1}{2} \\
 \text{for } \frac{1}{4} \quad | \quad 1 \quad 3 \quad 4\frac{1}{2}
 \end{array} \\
 \hline
 \pounds 1739 \quad 9 \quad 7\frac{1}{2} \text{ Ans.}
 \end{array}$$

CASE IX.

37 cwt. 2 qrs. 14 lb. at 7l. 10s. 9d. per cwt.

$$\begin{array}{r}
 \underline{6} \\
 45 \quad 4 \quad 6 \\
 \underline{6} \\
 271 \quad 7 \quad 0 \\
 \begin{array}{l}
 1 \text{ cwt.} \quad | \quad 7 \quad 10 \quad 9 \\
 2 \text{ qrs. is } \frac{1}{4} \quad | \quad 3 \quad 15 \quad 4\frac{1}{2} \\
 14 \text{ lb. is } \frac{1}{4} \quad | \quad 0 \quad 18 \quad 10
 \end{array} \\
 \hline
 \pounds 283 \quad 11 \quad 11\frac{1}{2} \text{ Ans.}
 \end{array}$$

17 cwt. 1 qr. 12 lb. at 1l. 19s. 8d. per cwt.

$$\begin{array}{r}
 \underline{4} \\
 7 \quad 18 \quad 8 \\
 \underline{4} \\
 31 \quad 14 \quad 8 \\
 \begin{array}{l}
 1 \text{ cwt.} \quad | \quad 1 \quad 19 \quad 8 \\
 1 \text{ qr. is } \frac{1}{4} \quad | \quad 0 \quad 9 \quad 11 \\
 7 \text{ lb. is } \frac{1}{4} \quad | \quad 0 \quad 2 \quad 5\frac{1}{2} \\
 4 \text{ lb. is } \frac{1}{4} \quad | \quad 0 \quad 1 \quad 5 \\
 1 \text{ lb. is } \frac{1}{4} \quad | \quad 0 \quad 0 \quad 4\frac{1}{2}
 \end{array} \\
 \hline
 \pounds 34 \quad 8 \quad 6 \text{ Ans.}
 \end{array}$$

23 cwt. 3 qrs. 8 lb. at 3*l*. 19*s*. 11*d*. per cwt.

		4
	15	19 8
		5
	79	18 4
3 cwt.	11	19 9
2 qr. is $\frac{1}{2}$	1	19 11 $\frac{1}{2}$
1 qr. is $\frac{1}{2}$	0	19 11 $\frac{1}{2}$
7 lb. is $\frac{1}{2}$	0	4 11 $\frac{1}{2}$
1 lb. is $\frac{1}{2}$	0	0 8 $\frac{1}{2}$
		£95 3 8 $\frac{1}{2}$ Ans.

39 cwt. 0 qr. 10 lb. at 1*l*. 17*s*. 10*d*. per cwt.

		6
	11	7 0
		6
	68	2 0
3 cwt.	5	13 6
8 lb. is $\frac{1}{4}$ cwt.	0	2 8 $\frac{1}{2}$
2 lb. is $\frac{1}{4}$	0	0 8
		£73 18 10 $\frac{1}{2}$ Ans.

PROMISCUOUS EXAMPLES, p. 66.

73 cwt. 1 qr. of sugar, at 3*l*. 15*s*. 7*d*. per cwt.

		9
	34	0 3
		8
	272	2 0
1 cwt.	3	15 7
1 qr. is $\frac{1}{4}$	0	18 10 $\frac{1}{2}$
		£276 16 5 $\frac{1}{2}$ Ans.

17 tons, 2 cwt. 3 qrs. 12 lb. at 9*l*. per ton.

		17
		153
2 cwt. is $\frac{1}{10}$ of a ton	0	18 0
2 qrs. is $\frac{1}{2}$	0	4 6
1 qr. is $\frac{1}{2}$	0	2 3
8 lb. is $\frac{1}{2}$ of 2 qrs.	0	0 7 $\frac{1}{2}$
4 lb. is $\frac{1}{2}$	0	0 3 $\frac{1}{2}$
		£154 5 8 $\frac{1}{2}$ Ans.

3 qrs. $12\frac{1}{2}$ lb. at $2l. 16s. 10d.$ per cwt.

2 qrs. is $\frac{1}{4}$	1	8	5
1 qr. is $\frac{1}{4}$	0	14	$2\frac{1}{2}$
8 lb. is $\frac{1}{4}$ of 2 qrs.	0	4	$0\frac{1}{2}$
4 lb. is $\frac{1}{4}$	0	2	$0\frac{1}{2}$
$\frac{1}{2}$ lb. is $\frac{1}{8}$	0	0	3
	<u>£2</u>	<u>8</u>	<u>$11\frac{1}{2}$</u>

Ans.

24 sacks, 9 tods, 1 stone of wool, at $2l. 10s. 6d.$ per sack.

			6
		15	3 0
			4
		60	12 0
$6\frac{1}{2}$ tods is $\frac{1}{4}$ a sack	1	5	3
3 tods is $\frac{1}{8}$	0	11	$7\frac{1}{2}$
	<u>£62</u>	<u>8</u>	<u>$10\frac{1}{2}$</u>

Ans.

125 yards, 3 qrs. of cloth, at $2s. 8\frac{1}{2}d.$ per yard.

			5
		13	$6\frac{1}{2}$ = price of 5 yards.
			5
		3	7 $8\frac{1}{2}$ = price of 25 yards.
			5
		16	18 $6\frac{1}{2}$ = price of 125 yards.
2 qrs. is $\frac{1}{4}$ yd.		1	$4\frac{1}{2}$
1 qr. is $\frac{1}{4}$ yd.		0	8
	<u>£17</u>	<u>0</u>	<u>$6\frac{1}{2}$</u>

Ans.

13 Eng. ells, 2 qrs. 2 nails, at $3s. 7\frac{1}{2}d.$ per ell.

			6
		1	1 9
			2
price of 12 ells	2	3	6
price of 1 ell	0	3	$7\frac{1}{2}$
1 qr. is $\frac{1}{4}$	0	0	$8\frac{1}{2}$
1 qr. is $\frac{1}{4}$	0	0	$8\frac{1}{2}$
2 nls. = $\frac{1}{4}$ qr.	0	0	$4\frac{1}{2}$
	<u>£2</u>	<u>8</u>	<u>$10\frac{1}{2}$</u>

Ans.

713 acres, 3 roods, 39 poles, at 3*l.* 17*s.* 6*d.* per acre.

			10
		38	15 0
			10
		387	10 0
			7
700 acres =	27	12	10 0
13 acres =	50	7	6
2 roods =	1	18	9
1 rood =	0	19	4½
20 poles =	0	9	8½
10 poles =	0	4	10
5 poles =	0	2	5
4 poles =	0	1	11½
	<u>£2766</u>	<u>14</u>	<u>6</u> Ans.

76 hhds. 1 tierce of wine, at 25*l.* 13*s.* 6*d.* per hhd.

			5
price of 5 hhds.	128	7	6
			5
price of 25 hhds.	641	17	6
			3
price of 75 hhds.	1925	12	6
1 tierce = $\frac{2}{3}$ hhd.	17	2	4
	<u>£1942</u>	<u>14</u>	<u>10</u> Ans.

24 gallons, 3 quarts of oil, at 3*s.* 4½*d.* per gallon.

			6
		1	0 3
			4
		4	1 0
2 qts. = $\frac{1}{2}$ gall.	0	1	8½
1 qt. is $\frac{1}{4}$ gall.	0	0	10
	<u>£4</u>	<u>3</u>	<u>6½</u> Ans.

57 hhds. 41 gallons of ale, at 4*l*. 10*s*. 6*d*. per hhd.

	8	
	<u>36</u>	4 0
		7
price of 56 hhds. =	<u>253</u>	8 0
price of 1 hhd. =		4 10 6
price of 57 hhds. =	<u>257</u>	18 6
27 galls. = $\frac{1}{4}$	2	5 3
9 galls. = $\frac{1}{4}$	0	15 1
4 $\frac{1}{2}$ galls. = $\frac{1}{2}$	0	7 6 $\frac{1}{2}$
$\frac{1}{2}$ gall. = $\frac{1}{2}$	0	0 10
	<u>£261</u>	7 2 $\frac{1}{2}$ Ans.

43 qrs. 5 bushels of wheat, at 1*l*. 8*s*. 6*d*. per qr.

	6	
	<u>8</u>	11 0
		7.
price of 42 qrs.	<u>59</u>	17 0
1 qr.		1 8 6
4 bush. is $\frac{1}{4}$	0	14 3
1 bush. is $\frac{1}{4}$	0	3 6 $\frac{1}{2}$
	<u>£62</u>	3 3 $\frac{1}{2}$ Ans.

Hire of coach and horses for 9 months, 11 days, at 5*l*. 10*s*. per month.

	5 <i>l</i> . 10 <i>s</i> .	
	9	
cost for 9 months	<u>49</u>	10
7 days = $\frac{1}{4}$ month	1	7 6
4 days = $\frac{1}{4}$ month	0	15 8 $\frac{1}{2}$
	<u>£51</u>	13 2 $\frac{1}{2}$ Ans.

TARE AND TRETT.

CASE I.

	(2)	
	<i>cwt.</i>	<i>gr.</i> <i>lb.</i>
lbs.	22	0 8 gross wt.
$10 \times 24 =$	2	0 16 tare.
	<u>19</u>	3 20 net wt.
	4	
	79	qrs.
	<u>28</u>	
	652	
	<u>158</u>	
	2232	lbs.

	(3)	
	<i>cwt.</i>	<i>gr.</i> <i>lb.</i>
	4	3 19
	<u>5</u>	
	24	2 11
	<u>9</u>	
	221	1 15 gross wt.
	<u>2</u>	1 14 tare.
	219	0 1 net wt.

	(4)	
	<i>cwt.</i>	<i>gr.</i> <i>lb.</i>
	45	1 17 gross wt.
	<u>2</u>	2 17 tare.
	42	3 0 net wt.

CASE II.

	(1)	
	<i>cwt.</i>	<i>gr.</i> <i>lb.</i>
	2	3 4
	<u>8</u>	
	22	1 4
	<u>2</u>	
	44	2 8
	<u>2</u>	3 4
	47	1 12 gross.
$17 \times 9 =$	1	1 13 tare.
	<u>45</u>	3 27 net.

	(2)	
	<i>cwt.</i>	<i>gr.</i> <i>lb.</i>
	0	3 19
	<u>2</u>	
	1	3 10
	<u>12</u>	
	22	0 8
	<u>10</u>	
	220	2 24
	<u>3</u>	19
	221	2 15 gross.
$10 \times 241 =$	21	2 2 tare.
	<u>200</u>	0 13 net.

CASE III.

(2)

201 lbs.					
<u>7</u>					
gross weight	1407	lbs.	=	cwt.	gr. lb.
				12	2 7
8 lb. is	$\frac{1}{14}$			0	3 $16\frac{1}{2}$
2 lb. is	$\frac{1}{7}$			0	0 25
				1	0 $13\frac{1}{2}$ tare.
				12	2 7 gross.
				11	1 $21\frac{1}{2}$ net.

or 1281 lbs. 6 oz. nearly.

(3)

cwt.	gr.	lb.	oz.		£.	s.	d.
4	3	17	0		1 qr. is $\frac{1}{4}$)	4	13 6
		<u>12</u>					8
58	3	8	0			37	8 0
4	3	17	0				7
lb.						261	16 0
8 is $\frac{1}{14}$	63	2	25 0 gross.			1	3 $4\frac{1}{2}$
4 is $\frac{1}{7}$	4	2	5 12		7 lb. is $\frac{1}{7}$	0	5 10
1 is $\frac{1}{14}$	2	1	2 14		1 lb. is $\frac{1}{7}$	0	0 10
	0	2	7 11		8 oz. is $\frac{1}{7}$	0	0 5
	7	1	16 5 tare.		2 oz. is $\frac{1}{7}$	0	0 $1\frac{1}{2}$
	56	1	8 11 net.		1 oz. is $\frac{1}{7}$	0	0 $0\frac{1}{2}$
						£263	6 $7\frac{1}{2}$

CASE IV.

(2)

cwt.	gr.	lb.	
152	1	3	gross.
8 lb. is $\frac{1}{14}$	10	3	14
2 lb. is $\frac{1}{7}$	2	2	24
	13	2	10 tare.
26)	138	2	21 suttie
	5	1	9 trett.
	133	1	12 net.

(3)

cwt.	gr.	lb.	
3	1	5	
		<u>7</u>	
	23	0	7 gross.
14 lb. is $\frac{1}{7}$	2	3	15 nearly.
2 lb. is $\frac{1}{7}$	0	1	18
1 lb. is $\frac{1}{7}$	0	0	23
$\frac{1}{2}$ lb. is $\frac{1}{7}$	0	0	$11\frac{1}{2}$
	3	2	$11\frac{1}{2}$ tare.
26)	19	1	$23\frac{1}{2}$ suttie
	0	2	27 trett.
	18	2	$24\frac{1}{2}$ net.

CASE V.

(2)

<i>cwt.</i>	<i>qr.</i>	<i>lb.</i>		<i>cwt.</i>	<i>qr.</i>	<i>lb.</i>
13	1	17		215	0	16 net wt.
		2				4
26	3	6				860
		9				28
241	0	26		24096	lbs. at	5½ <i>d.</i>
13	1	17				5
254	2	15 gross.		120480		
8 lb. is ¼	18	0 21		1 <i>d.</i> is ¼	12048	
4 lb. is ½	9	0 10½		½ <i>d.</i> is ½	6024	
1 lb. is 1	2	1 2½		12)	138552	
	29	2 6 tare.		2,0)	1154,6	
26)	225	0 9*			£577 6 value req.	
	8	2 17 trett.				
168)	216	1 20uttle.				
	1	1 4 cloff.				
	215	0 16 net wt.				

(3)

<i>cwt.</i>	<i>qr.</i>	<i>lb.</i>		<i>£.</i>	<i>s.</i>	<i>d.</i>
3	0	14		1	11	6
		4				80
12	2	0		126	0	0
		7		1 qr. is ¼	0	7 10½
87	2	0		14 lb. is ½	0	3 11½
3	0	14		7 lb. is ½	0	1 11½
90	2	14 gross.		1 lb. is ¼	0	0 3½
8 lb. is ¼	6	1 25 tare.		£126	14	0½ value req.
26)	84	0 17				
	3	0 26 trett.				
168)	80	3 19				
	0	1 25 cloff.				
	80	1 22 net wt.				

* This number is obtained by subtracting the tare from the gross

(4)

cwt.	qr.	lb.		£.	s.	d.
5	2	25		8	10	6
		5				5
28	2	13	gross.	42	12	6
8 lb. is $\frac{1}{4}$	2	0	5 tare.			5
26)	26	2	8	213	2	6
	1	0	2 trett.	1 qr. is $\frac{1}{4}$	2	2
	25	2	6	14 lb. is $\frac{1}{2}$	1	1
			10 cloff.	7 lb. is $\frac{1}{2}$	0	10
	25	1	24 net wt.	2 lb. is $\frac{1}{2}$ of 14 lb.	0	3
				1 lb. is $\frac{1}{2}$	0	1
				value = £217	1	7 $\frac{1}{2}$

VULGAR FRACTIONS.

CASE I.—RULE I.

(2) $\frac{4}{3} \div \frac{3}{7} = \frac{1}{3} \div \frac{1}{7} = \frac{7}{3}$ Ans.

RULE II. p. 72.

(2)

$$\begin{array}{r} 48 \overline{)272(6} \\ \underline{288} \\ \text{G. C. M.} = 16 \end{array}$$

$$16 \overline{)48(3}$$

$$16) \frac{4}{3} \div \frac{3}{7} = \frac{7}{3} \text{ Ans.}$$

(4)

$$\begin{array}{r} 825 \overline{)960(1} \\ \underline{825} \\ 135 \end{array}$$

$$135 \overline{)825(6}$$

$$135 \overline{)135(9}$$

$$15) \frac{8}{3} \div \frac{3}{4} = \frac{4}{3} \text{ Ans.}$$

(3)

$$\begin{array}{r} \text{G. C. M.} = 192 \overline{)576(3} \\ \underline{576} \\ 192 \end{array}$$

$$192) \frac{1}{3} \div \frac{3}{8} = \frac{8}{3} \text{ Ans.}$$

(5)

$$\begin{array}{r} 252 \overline{)364(1} \\ \underline{252} \\ 112 \end{array}$$

$$112 \overline{)252(2}$$

$$112 \overline{)112(4}$$

$$28) \frac{8}{3} \div \frac{3}{4} = \frac{4}{3} \text{ Ans.}$$

(6)

$$5184 \overline{) 6012(1}$$

$$\underline{5184}$$

$$828 \overline{) 5184(6}$$

$$\underline{4968}$$

$$216 \overline{) 828(4}$$

$$\underline{864}$$

$$\text{G. C. M.} = 36 \overline{) 216(6}$$

$$\underline{216}$$

$$36) \frac{1111}{1111} = 1 \frac{1}{3} \text{ Ans.}$$

(7)

$$1344 \overline{) 1536(1}$$

$$\underline{1344}$$

$$\text{G.C.M.} = 192 \overline{) 1344(7}$$

$$\underline{1344}$$

$$192) \frac{1111}{1111} = 7 \text{ Ans.}$$

(8)

$$42237 \overline{) 75582(2}$$

$$\underline{84474}$$

$$8892 \overline{) 42237(5}$$

$$\underline{44460}$$

$$\text{G. C. M.} = 2223 \overline{) 8892(4}$$

$$\underline{8892}$$

$$2223) \frac{111111}{111111} = 1 \frac{1}{3} \text{ Ans.}$$

CASE II.

$$22\frac{1}{3} = \frac{22 \times 5 + 1}{5} = 1\frac{1}{3} \text{ Ans.}$$

$$514\frac{5}{8} = \frac{514 \times 16 + 5}{16} = 2\frac{111}{8}$$

(5)

$$100\frac{1}{3} = \frac{100 \times 59 + 19}{59} = 2\frac{11}{3} \text{ Ans.}$$

(6)

$$47\frac{111}{111} = \frac{47 \times 8400 + 3147}{8400} = 2\frac{2747}{8400} \text{ Ans.}$$

(7)

$$296\frac{1}{3} = \frac{296 \times 3 + 2}{3} = 2\frac{2}{3} \text{ Ans.}$$

CASE III.

- (2) $\frac{49}{7} = 49 \div 7 = 7$ Ans.
 (3) $\frac{1245}{22} = 1245 \div 22 = 56\frac{13}{22}$ Ans.
 (4) $\frac{3848}{21} = 3848 \div 21 = 183\frac{5}{21}$ Ans.
 (5) $\frac{10101}{19} = 10101 \div 19 = 531\frac{12}{19}$ Ans.
 (6) $\frac{60162}{1524} = 60162 \div 1524 = 39\frac{726}{1524}, 39\frac{121}{254}$ Ans.

CASE IV.

$$\frac{4}{7} \times \frac{8}{9} = \frac{4 \times 8}{7 \times 9} = \frac{32}{63} \text{ Ans.} \quad \left| \quad \frac{2}{3} \times \frac{3}{5} \times \frac{5}{8} = \frac{2 \times 3 \times 5}{3 \times 5 \times 8} = \frac{1}{4} = \frac{1}{4}$$

$$\frac{3}{4} \times \frac{5}{8} \times \frac{1}{11} = \frac{3 \times 5 \times 1}{4 \times 8 \times 11} = \frac{15}{352} \text{ Ans.}$$

$$\frac{1}{12} \times \frac{7}{13} \times \frac{8}{19} \text{ of } 10 = \frac{11 \times 7 \times 8 \times 10}{12 \times 13 \times 19} = \frac{6160}{3042} = \frac{3080}{1521} \text{ Ans.}$$

$$\frac{2}{7} \times 9\frac{1}{2} = \frac{2}{7} \times \frac{19}{2} = \frac{2 \times 19}{7 \times 2} = \frac{19}{7} = 2\frac{5}{7} \text{ Ans.}$$

$$\frac{1}{3} \times 3\frac{1}{2} \times 4\frac{1}{5} = \frac{1}{3} \times \frac{7}{2} \times \frac{21}{5} = \frac{7 \times 21}{3 \times 5} = \frac{7 \times 7}{5} = \frac{49}{5} = 9\frac{4}{5} \text{ Ans.}$$

CASE V.

$$\frac{2}{9} \text{ and } \frac{4}{7} = \frac{2 \times 7}{9 \times 7} \text{ and } \frac{4 \times 9}{7 \times 9} = \frac{14}{63} \text{ and } \frac{36}{63}.$$

(3)
 Since 6 is a multiple of 2 and 3, the least common denominator of the four fractions will be the least common multiple of 6 and 8. Now 6 and 8 are both divisible by 2, and the quotients are 3 and 4; hence $2 \times 3 \times 4 = 24 =$ the least common denominator: then

$$\left. \begin{array}{l} 24 \div 2 = 12, \text{ and } \frac{1}{2} \times \frac{12}{12} = \frac{6}{6} \\ 24 \div 3 = 8, \text{ and } \frac{2}{3} \times \frac{8}{8} = \frac{16}{24} \\ 24 \div 6 = 4, \text{ and } \frac{3}{6} \times \frac{4}{4} = \frac{12}{24} \\ 24 \div 8 = 3, \text{ and } \frac{4}{8} \times \frac{3}{3} = \frac{12}{24} \end{array} \right\} \text{ Fractions required.}$$

(4)

$\frac{3}{4}$ of $\frac{4}{5} = \frac{3 \times 4}{4 \times 5} = \frac{3}{5}$; and $5\frac{1}{2} = \frac{11}{2}$; hence the fractions are $\frac{1}{2}$, $\frac{3}{5}$, $\frac{11}{2}$, and $\frac{1}{5}$: therefore

$$\left. \begin{array}{l} 1 \times 5 \times 2 \times 19 = 190 \\ 3 \times 3 \times 2 \times 19 = 342 \\ 11 \times 3 \times 5 \times 19 = 3135 \\ 2 \times 3 \times 5 \times 2 = 60 \\ 3 \times 5 \times 2 \times 19 = 570 \end{array} \right\} \begin{array}{l} \text{Therefore the required fractions} \\ \text{are} \\ \frac{190}{3135}, \frac{342}{3135}, \frac{11 \times 190}{3135}, \frac{60}{3135}. \end{array}$$

(5)

$\frac{3}{4}$ of $1\frac{1}{2} = \frac{3}{4}$ of $\frac{3}{2} = \frac{3 \times 3}{4 \times 2} = \frac{9}{8}$; hence the fractions are $1\frac{1}{2}$, $\frac{9}{8}$, $1\frac{1}{2}$, and $\frac{7}{8}$: therefore

$$\left. \begin{array}{l} 11 \times 16 \times 11 \times 7 = 13552 \\ 15 \times 13 \times 11 \times 7 = 15015 \\ 9 \times 13 \times 16 \times 7 = 13014 \\ 5 \times 13 \times 16 \times 11 = 11440 \\ 13 \times 16 \times 11 \times 7 = 16016 \end{array} \right\} \begin{array}{l} \text{The fractions required are} \\ \frac{13552}{11440}, \frac{15015}{11440}, \frac{13014}{11440}, \frac{16016}{11440}. \end{array}$$

(6)

$\frac{3}{4}$ of $4\frac{1}{2} = \frac{3}{4}$ of $\frac{9}{2} = \frac{27}{8}$; $\frac{7}{8}$ of $\frac{3}{4} = \frac{21}{32}$, and $\frac{1}{2}$ of $2\frac{1}{2} = \frac{1}{2}$ of $\frac{5}{2} = \frac{5}{4}$; hence the fractions are $\frac{27}{8}$, $\frac{21}{32}$, and $\frac{5}{4}$. Since 5 is contained in 20, the least common denominator is $20 \times 21 = 420$: then

$$\left. \begin{array}{l} 420 \div 5 = 84, \text{ and } \frac{5}{4} \times \frac{84}{21} = \frac{7 \times 84}{4 \times 21} \\ 420 \div 21 = 20, \text{ and } \frac{21}{32} \times \frac{20}{5} = \frac{4 \times 20}{8 \times 5} \\ 420 \div 20 = 21, \text{ and } \frac{27}{8} \times \frac{21}{3} = \frac{7 \times 21}{4 \times 3} \end{array} \right\} \text{The fractions required.}$$

(7)

The fractions are $\frac{3}{5}$, $\frac{7}{8}$, $\frac{11}{2}$, and 9 is a multiple of 3; hence $5 \times 9 = 45 =$ least common denominator: then

$$\left. \begin{array}{l} 45 \div 5 = 9, \text{ and } \frac{3}{5} \times \frac{9}{1} = \frac{27}{5} \\ 45 \div 9 = 5, \text{ and } \frac{7}{8} \times \frac{5}{1} = \frac{35}{8} \\ 45 \div 3 = 15, \text{ and } \frac{11}{2} \times \frac{15}{3} = \frac{165}{2} \\ 45 \div 1 = 45, \text{ and } \frac{1}{1} \times \frac{45}{1} = \frac{45}{1} \end{array} \right\} \text{The fractions required.}$$

CASE VI., p. 76.

(5)

$$\frac{3}{4} \text{ of } 1d. = \frac{3}{4} \text{ of } \frac{1}{12}s. = \frac{3}{4} \text{ of } \frac{1}{12} \text{ of } \frac{1}{20}l. = \frac{1}{160}l. = \frac{1}{2} \frac{1}{80}l.$$

(6)

$$\text{of } \frac{1}{2}d. = \frac{1}{2} \text{ of } \frac{1}{4} \text{ of } \frac{1}{12}s. = \frac{1}{2} \text{ of } \frac{1}{4} \text{ of } \frac{1}{12} \text{ of } \frac{1}{20}l. = \frac{1}{160}l.$$

$$(7) \\ \frac{1}{16}l. = \frac{1}{16} \times 20s. = \frac{1}{16} \times 20 \times 12d. = \frac{15}{2}d.$$

$$(8) \\ \frac{1}{2} \text{ of } 1 \text{ dwt.} = \frac{1}{2} \text{ of } \frac{1}{16} \text{ oz.} = \frac{1}{2} \text{ of } \frac{1}{16} \text{ of } \frac{1}{16} \text{ lb.} = \frac{1}{512} \text{ lb.}$$

$$(9) \\ \frac{1}{2} \text{ lb.} = \frac{1}{2} \text{ of } \frac{1}{16} \text{ cwt.} = \frac{1}{32} = \frac{1}{32} \text{ cwt.}$$

$$(10) \\ \frac{1}{16} \text{ hhd.} = \frac{1}{16} \text{ of } 63 \text{ gall.} = \frac{1}{16} \text{ of } 63 \text{ of } 8 \text{ pints} = \frac{3}{8} \text{ pint.}$$

$$(11) \\ \frac{1}{12} \text{ of } 1 \text{ month} = \frac{1}{12} \text{ of } 28 \text{ days} = \frac{7}{3} \text{ days.}$$

$$(12) \\ 7s. \text{ } 3d. = 87d.; \text{ also } 1l. = 240d.; \therefore \text{fraction} = \frac{87}{240} = \frac{29}{80}l.$$

$$(13) \\ 6 \text{ fur. } 16 \text{ po.} = 256 \text{ poles; also } 1 \text{ mile} = 320 \text{ poles; } \therefore \text{fraction required} = \frac{256}{320} = \frac{4}{5} \text{ mile.}$$

$$(14) \\ \frac{1}{2}l. = \frac{1}{2} \text{ of } 20s. = \frac{10}{1}s. = \frac{10}{1} \text{ of } \frac{1}{1} \text{ guinea} = \frac{10}{1} \text{ guinea.}$$

$$(15) \\ \frac{1}{2} \text{ cr.} = \frac{1}{2} \text{ of } 5s. = \frac{5}{2}s. = \frac{5}{2} \text{ of } \frac{1}{1} \text{ guinea} = \frac{5}{2} \text{ guinea.}$$

$$(16) \\ \frac{1}{2} \text{ half-cr.} = \frac{1}{2} \times 2\frac{1}{2}s. = \frac{1}{2} \times \frac{5}{2}s. = \frac{5}{4}s. = \frac{1}{4} \text{ shilling.}$$

$$(17) \\ \frac{1}{2} \text{ moi.} = \frac{1}{2} \times \frac{1}{2}s. = \frac{1}{4} \times \frac{1}{2}s. = \frac{1}{8}s. = \frac{1}{8} \text{ crown.}$$

$$(18) \\ 3 \text{ fur. } 17 \text{ po. } 0 \text{ yd. } 2 \text{ ft. } 4\frac{1}{2} \text{ in.} = 27154\frac{1}{2} \text{ in.} = \frac{108618}{1} \text{ in.,} \\ \text{and } 1 \text{ mile} = 1760 \text{ yds.} = 5280 \text{ ft.} = 63360 \text{ inches; } \therefore \\ \frac{108618}{63360} \times \frac{1}{16} \text{ mi.} = \frac{1}{16} \text{ mile.}$$

CASE VII., pp. 76, 77.

$$(2) \\ \frac{1}{2}l. = \frac{1}{2} \times 20s. = \frac{10}{1}s. = 10s. \text{ } 12d. \text{ } \frac{1}{2}$$

$$(3) \\ \frac{1}{2} \text{ gui.} = \frac{2 \times 21}{9}s. = \frac{2 \times 7}{3}s. = \frac{14}{3}s. = 4s. \text{ } 8d.$$

(4)

$$\frac{4}{7} \text{ cr.} = \frac{4 \times 5}{7} s. = \frac{20}{7} s. = 2s. 10\frac{1}{2}d. \frac{1}{7}$$

(5)

$$\frac{13}{19} \text{ moi.} = \frac{13 \times 27}{19} s. = \frac{351}{19} s. = 18s. 5\frac{1}{2}d. \frac{1}{19}$$

(6)

$$\frac{3}{5} \text{ lb.} = \frac{3 \times 12}{5} \text{ oz.} = \frac{36}{5} \text{ oz.} = 7 \text{ oz. } 4 \text{ dwt.}$$

(7)

$$\frac{4}{7} \text{ lb.} = \frac{4 \times 16}{7} \text{ oz.} = \frac{64}{7} \text{ oz.} = 9 \text{ oz. } 2\frac{2}{7} \text{ dr.}$$

(8)

$$\frac{5}{9} \text{ ell E.} = \frac{5 \times 5}{9} \text{ qr.} = \frac{25}{9} \text{ qr.} = 2 \text{ qrs. } 3\frac{1}{3} \text{ nl.}$$

(9)

$$\frac{5}{8} \text{ ac.} = \frac{5 \times 4}{8} \text{ ro.} = \frac{20}{8} \text{ ro.} = 2 \text{ ro. } 20 \text{ po.}$$

(10)

$$\frac{2}{15} \text{ hhd.} = \frac{2 \times 54}{15} \text{ galls.} = \frac{2 \times 18}{5} \text{ galls.} = \frac{36}{5} \text{ galls.} = 7 \text{ galls. } 0 \text{ qt. } 1\frac{2}{5} \text{ pt.}$$

(11)

$$\frac{7}{8} \text{ tun} = \frac{7 \times 4}{8} \text{ hhds.} = \frac{28}{8} \text{ hhds.} = 3 \text{ hhds. } 31 \text{ galls. } 2 \text{ qts.}$$

(12)

$$\frac{7}{9} \text{ cwt.} = \frac{7 \times 4}{9} \text{ qrs.} = \frac{28}{9} \text{ qrs.} = 3 \text{ qrs. } 3 \text{ lbs. } 1 \text{ oz. } 12\frac{4}{9} \text{ drs.}$$

(13)

$$\frac{5}{9} \text{ qr.} = \frac{5 \times 8}{9} \text{ bush.} = \frac{40}{9} \text{ bush.} = 4 \text{ bush. } 1 \text{ pk. } 1 \text{ gall. } 2\frac{2}{9} \text{ qts.}$$

(14)

$$\frac{7}{13} \text{ day} = \frac{7 \times 24}{13} \text{ hrs.} = \frac{168}{13} \text{ hrs.} = 12 \text{ hrs. } 55 \text{ min. } 23\frac{1}{13} \text{ sec.}$$

$$\begin{aligned} & \text{(15)} \\ \frac{3}{10} \text{ mi.} &= \frac{35 \times 8}{108} \text{ fur.} = \frac{35 \times 2}{27} \text{ fur.} = 1 \frac{1}{3} \text{ fur.} = \\ & 2 \text{ fur. } 23 \text{ po. } 3 \text{ yds. } 2 \text{ ft. } 7 \frac{1}{2} \text{ in.} \end{aligned}$$

ADDITION OF FRACTIONS.

$$\begin{aligned} & \text{(4)} \\ \frac{1}{3} \text{ of } \frac{2}{3} &= \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}; \text{ then} \\ \frac{1}{3} + \frac{1}{3} + \frac{1}{3} &= \frac{1}{3} + \frac{2}{9} + \frac{2}{9} = \frac{5+4+2}{9} = \frac{11}{9} = 1 \frac{2}{9}; \text{ and } 7 + 1 \frac{2}{9} = 8 \frac{2}{9}. \end{aligned}$$

$$\begin{aligned} & \text{(5)} \\ \frac{1}{2} \text{ of } \frac{1}{3} &= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}; \text{ then} \\ \frac{1}{2} + \frac{1}{3} + \frac{1}{6} &= \frac{3}{6} + \frac{2}{6} + \frac{1}{6} = \frac{3+2+1}{6} = \frac{6}{6} = 1; \text{ and} \\ 9 + 1 &= 10. \end{aligned}$$

$$\begin{aligned} & \text{(6)} \\ \frac{1}{10} \text{ of } 6 \frac{7}{8} &= \frac{1}{10} \times \frac{55}{8} = \frac{9 \times 11}{2 \times 8} = 1 \frac{1}{8} = 6 \frac{9}{8}; \\ \frac{1}{2} \text{ of } \frac{1}{2} &= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}; \text{ then} \\ \frac{1}{2} + \frac{1}{2} + \frac{1}{2} &= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} = 1 \frac{1}{2}; \\ \text{and } 6 + 7 + 1 \frac{1}{2} &= 13 \frac{1}{2} = \text{sum.} \end{aligned}$$

$$\begin{aligned} & \text{(7)} \\ \frac{1}{2} + \frac{1}{3} + \frac{1}{6} \text{ of } \frac{1}{2} &= \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{40+96+45}{120} = 1 \frac{1}{120}; \\ \text{and } 2 + 1 \frac{1}{120} &= 3 \frac{1}{120} = \text{sum.} \end{aligned}$$

$$\begin{aligned} & \text{(8)} \\ \frac{1}{2} \text{ of } 5s. 6d. &= \frac{16s. 6d.}{4} = 0 \quad 4 \quad 1 \frac{1}{2} \\ \frac{1}{3} \text{ of } \frac{1}{2} \text{ of } 17s. 2 \frac{1}{2}d. &= \frac{34s. 4 \frac{1}{2}d.}{15} = 0 \quad 2 \quad 3 \frac{1}{2} \\ \frac{1}{4} \text{ of } 5l. 6s. 7 \frac{1}{2}d. &= \frac{26l. 13s. 0 \frac{1}{2}d.}{7} = 3 \quad 16 \quad 1 \frac{1}{2} \\ \text{Sum} &= \underline{\underline{\pounds 4 \quad 2 \quad 6 \frac{1}{2}}} \end{aligned}$$

(9)

	<i>£.</i>	<i>s.</i>	<i>d.</i>
$\frac{2}{7}$ of 15 <i>l.</i>	= $\frac{30}{7}$ <i>l.</i> = 4	5	$8\frac{1}{2}$ $\frac{2}{7}$
3 <i>l.</i>	= $\frac{21}{7}$ <i>l.</i> = 3	8	$6\frac{1}{2}$ $\frac{2}{7}$
$\frac{1}{2}$ of $\frac{2}{7}$ of $\frac{3}{4}$ of 1 <i>l.</i>	= $\frac{1}{7}$ <i>l.</i> = 0	2	$10\frac{1}{2}$ $\frac{1}{7}$
$\frac{2}{3}$ of $\frac{2}{7}$ of 1 <i>s.</i>	= $\frac{2}{7}$ <i>s.</i> = 0	0	$3\frac{1}{2}$ $\frac{2}{7}$
Sum =	<u>£7</u>	<u>17</u>	<u>$5\frac{1}{2}$ $\frac{2}{7}$</u>

(10)

	<i>yd.</i>	<i>yd.</i>
$\frac{2}{3}$ yd. =	= $\frac{2}{3}$ =	$1\frac{2}{3}$
$\frac{1}{2}$ ft. = $\frac{1}{2} \times \frac{1}{2}$ yd. = $\frac{1}{4}$ yd.	= $\frac{1}{4}$ =	$1\frac{3}{4}$
$\frac{3}{8}$ mile = $\frac{3 \times 1760}{8}$ yds.	= 3×220 =	660
Sum =	<u>$660\frac{1}{4}$</u>	<u>yd.</u>

(11)

	<i>days.</i>	<i>hrs.</i>
$\frac{1}{2}$ week = $\frac{7}{2}$ days	= 2	8
$\frac{1}{4}$ day = $\frac{24}{4}$ hrs.	= 0	6
$\frac{1}{2}$ hr. =	.	$\frac{1}{2}$
Sum =	<u>2</u>	<u>$14\frac{1}{2}$</u>

(12)

$$\frac{1}{2} + \frac{2}{3} \text{ of } \frac{3}{4} = \frac{1}{2} + \frac{1}{4} = \frac{2+1}{4} = \frac{3}{4}; \text{ then } 4 + 3 + \frac{7}{6} = 7\frac{7}{6} = \text{sum.}$$

(13)

	<i>£.</i>	<i>s.</i>	<i>d.</i>
$\frac{2}{3}$ gui. = $\frac{5 \times 21}{9}$ s. = $\frac{5 \times 7}{3}$ s. = $\frac{35}{3}$ s.	= 0	11	8
$\frac{3}{8}$ moi. = $\frac{3 \times 27}{8}$ s. = $\frac{81}{8}$ s.	= 0	10	$1\frac{1}{2}$
Sum =	<u>£1</u>	<u>1</u>	<u>$9\frac{1}{2}$</u>

(14)

	<i>qr.</i>	<i>lb.</i>	<i>oz.</i>
$\frac{4}{7}$ cwt. = $\frac{4 \times 4}{7}$ qrs. = $\frac{16}{7}$ qrs.	= 2	8	0
$8\frac{1}{2}$ lb. = 8 lbs. + $\frac{5 \times 16}{6}$ oz.	= 0	8	$13\frac{1}{2}$
Sum =	<u>2</u>	<u>17</u>	<u>$1\frac{7}{6}$</u>

(15)

$$\begin{array}{rcl}
 3\frac{1}{2} \text{ ells E.} & = & 17\frac{1}{2} \text{ qrs.} \\
 4\frac{1}{2} \text{ yds.} & = & . \quad . \quad .
 \end{array}
 \begin{array}{rcl}
 & \text{yd.} & \text{qr.} \text{ nl.} \\
 & 4 & 1 \quad 2 \\
 & 4 & 2 \quad 0 \\
 & 0 & 0 \quad 0\frac{1}{2} \\
 \hline
 \text{Sum} & = & 8 \quad 3 \quad 2\frac{1}{2}
 \end{array}$$

(16)

$$\begin{array}{rcl}
 \frac{3}{4} \text{ hhd.} & = \frac{3 \times 54}{4} \text{ galls.} & = \frac{3 \times 27}{2} \text{ galls.} \\
 \frac{1}{2} \text{ gall.} & = \frac{5 \times 4}{7} \text{ qts.} & = \frac{2}{7} \text{ qts.} \\
 \frac{1}{2} \text{ of } \frac{1}{2} \text{ of 1 pt.} & = \frac{1}{2} \text{ pt.} & = . \quad . \quad .
 \end{array}
 \begin{array}{rcl}
 & \text{gall.} & \text{qt.} \text{ pt.} \\
 & 40 & 2 \quad 0 \\
 & 0 & 2 \quad 1\frac{1}{2} \\
 & 0 & 0 \quad 0\frac{1}{2} \\
 \hline
 \text{Sum} & = & 41 \quad 0 \quad 2\frac{1}{2}
 \end{array}$$

(17)

$$\begin{array}{rcl}
 \frac{1}{2} \text{ of } \frac{1}{2} \text{ of } 7\frac{1}{2} \text{ lb.} & = \frac{1}{2} \times \frac{1}{2} \times 7\frac{1}{2} \times \frac{1}{2} \text{ oz.} & = 26\frac{1}{8} \text{ oz.} \\
 \frac{1}{2} \text{ of } \frac{1}{2} \text{ of } 5\frac{1}{2} \text{ oz.} & = \frac{1}{2} \times \frac{1}{2} \times 5\frac{1}{2} \times \frac{1}{2} \text{ oz.} & = 2\frac{1}{8} \text{ oz.} \\
 \hline
 \text{Sum} & = & 28\frac{1}{4} \text{ oz.}
 \end{array}$$

SUBTRACTION OF FRACTIONS, p. 80.

(5)

$$\frac{679}{700} - \frac{300}{700} = \frac{679-300}{700} = \frac{379}{700}$$

(6)

$$69\frac{1}{2} - 14\frac{1}{2} = 68\frac{1}{2} - 14\frac{1}{2} = 68\frac{1}{2} - 14\frac{1}{2} = 54\frac{1}{2}$$

(7)

$$14\frac{1}{2} - \frac{1}{2} \text{ of } 19 = 14\frac{1}{2} - 12\frac{1}{2} = 13\frac{1}{2} - 12\frac{1}{2} = 1\frac{1}{2}$$

(8)

$$\begin{array}{rcl}
 \frac{1}{2} \text{ l.} & = & 10 \quad 0 \\
 \frac{1}{2} \text{ s.} & = & 0 \quad 9 \\
 \hline
 \text{Diff.} & = & 9 \quad 3
 \end{array}$$

(9)

$$\begin{array}{rcl}
 \frac{1}{2} \text{ oz.} & = \frac{3 \times 20}{5} \text{ dwt.} & = 12 \quad 0 \\
 \frac{1}{2} \text{ dwt.} & = \frac{7 \times 24}{8} \text{ gr.} & = 0 \quad 21 \\
 \hline
 \text{Diff.} & = & 11 \quad 3
 \end{array}$$

H 2

(10)

$$\frac{2}{3} \text{ league} = \frac{2 \times 3}{3} \text{ miles} . \quad \begin{array}{c} \text{mi. fur. po.} \\ = 2 \quad 0 \quad 0 \end{array}$$

$$\frac{7}{10} \text{ mile} = \frac{7 \times 8}{10} \text{ fur.} = \frac{56}{10} \text{ fur.} = 5 \frac{6}{10} \text{ fur.} = 5 \frac{3}{5} \text{ fur.} = 0 \quad 5 \quad 24$$

$$\text{Diff.} = \frac{1 \quad 2 \quad 16}{10}$$

(11)

$$\begin{array}{c} \text{wk. da. hr. mi.} \\ 9 \frac{7}{10} \text{ days} = 1 \quad 2 \quad 16 \quad 48 \\ \text{Diff.} = \frac{5 \quad 4 \quad 7 \quad 12}{10} \end{array}$$

(12)

$$\begin{array}{c} \text{cwt. gr. lb.} \\ 4 \frac{7}{10} \text{ cwt.} = 4 \quad 1 \quad 20 \\ 14 \frac{9}{10} \text{ lbs.} = 0 \quad 0 \quad 14 \frac{9}{10} \\ \text{Diff.} = \frac{4 \quad 1 \quad 5 \frac{1}{10}}{10} \end{array}$$

(13)

$$\begin{array}{l} 100 \frac{7}{8} = 100 \frac{10}{8} \\ \frac{7}{8} \text{ of } 10 = \frac{7 \frac{7}{8}}{1 \frac{1}{4}} \\ \text{Diff.} = \frac{93 \frac{3}{4}}{1 \frac{1}{4}} \end{array}$$

(14)

$$\begin{array}{c} \text{cwt. gr. lb. oz. dr.} \\ 8 \frac{7}{10} \text{ cwt.} = 8 \quad 2 \quad 22 \quad 6 \quad 6 \frac{2}{5} \\ 2 \text{ qrs. } 3 \frac{7}{8} \text{ lbs.} = 0 \quad 2 \quad 3 \quad 9 \quad 2 \frac{7}{8} \\ \text{Diff.} = \frac{8 \quad 0 \quad 18 \quad 13 \quad 4 \frac{4}{5}}{10} \end{array}$$

MULTIPLICATION OF FRACTIONS, p. 81.

(3)

$$\frac{4}{15} \times \frac{5}{24} = \frac{4 \times 5}{15 \times 24} = \frac{1 \times 1}{3 \times 6} = \frac{1}{18}.$$

(4)

$$4 \frac{1}{2} \times \frac{1}{2} = \frac{9}{2} \times \frac{1}{2} = \frac{9 \times 1}{2 \times 2} = \frac{9}{4}.$$

(5)

$$\frac{1}{2} \text{ of } 7 \times \frac{1}{2} = \frac{7}{2} \times \frac{1}{2} = \frac{7 \times 1}{2 \times 2} = \frac{7}{4} = 1 \frac{3}{4}.$$

(6)

$$\frac{2}{3} \text{ of } \frac{1}{2} \times \frac{1}{2} \text{ of } 3 \frac{1}{2} = \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2} \times \frac{7}{2} = \frac{1 \times 1 \times 1 \times 23}{3 \times 1 \times 4 \times 7} = \frac{23}{84}.$$

(7)

$$\begin{aligned} 4 \frac{1}{2} \times \frac{3}{4} \text{ of } \frac{1}{2} \times 18 \frac{1}{2} &= \frac{9}{2} \times \frac{3}{4} \times \frac{1}{2} \times \frac{37}{2} = \frac{9 \times 3 \times 1 \times 47}{1 \times 4 \times 7 \times 5} \\ &= \frac{1 \frac{3}{4} \frac{6}{5}}{1 \frac{4}{5}} = 9 \frac{9}{10}. \end{aligned}$$

(8)

$$\frac{2}{3} \times 3\frac{1}{2} \times 5 \times \frac{2}{3} \text{ of } \frac{1}{3} = \frac{2}{3} \times \frac{1^2}{4} \times \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{1 \times 13 \times 1 \times 1 \times 3}{1 \times 2 \times 1 \times 4 \times 1} = \frac{3^2}{4} = 4\frac{3}{4}.$$

(9)

$$\frac{1}{2} \times \frac{2}{3} \times \frac{2}{7} \times \frac{2}{3} \times \frac{2^2}{3} = \frac{1 \times 1 \times 2 \times 1 \times 25}{1 \times 1 \times 7 \times 1 \times 3} = \frac{50}{21} = 2\frac{8}{21}.$$

(10)

$$\frac{1^4}{1} \times \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{2 \times 1 \times 2 \times 9 \times 44}{1 \times 3 \times 1 \times 1 \times 1} = 2 \times 2 \times 3 \times 44 = 528.$$

(11)

$$\frac{1}{2} \times \frac{1}{2} \times \frac{2}{3} \times \frac{1}{2} = \frac{11 \times 11}{5 \times 9 \times 6} = \frac{121}{27}.$$

(12)

$$\frac{7}{3} \times \frac{17}{4} \times \frac{2^2}{3} = \frac{7 \times 17 \times 13}{3 \times 2 \times 5} = \frac{1547}{30} = 51\frac{17}{30}.$$

DIVISION OF FRACTIONS, p. 81.

(3)

$$\frac{4}{7} \div \frac{2}{3} = \frac{4}{7} \times \frac{3}{2} = \frac{2 \times 3}{7 \times 1} = \frac{6}{7}.$$

(4)

$$\frac{5^2}{6} \div \frac{7}{2} = \frac{5^2}{6} \times \frac{2}{7} = \frac{55}{3 \times 7} = \frac{55}{21} = 2\frac{13}{21}.$$

(5)

$$\frac{1^2}{6} \div \frac{1^2}{2} = \frac{1^2}{6} \times \frac{2}{1^2} = \frac{2}{3} = \frac{1}{3}.$$

(6)

$$\frac{7}{8} \div \frac{1}{2} = \frac{7}{8} \times \frac{2}{1} = \frac{7}{4}.$$

(7)

$$\frac{1}{2} \text{ of } \frac{2}{3} \div \frac{2}{3} \text{ of } \frac{2}{3} = \frac{1}{2} \div \frac{1}{2} = \frac{1}{2} \times \frac{2}{1} = \frac{2}{2}.$$

(8)

$$5 \div \frac{7}{10} = \frac{5}{1} \times \frac{10}{7} = \frac{50}{7} = 7\frac{1}{7}.$$

(9)

$$5205\frac{1}{2} \div \frac{1}{2} \text{ of } 91 = \frac{2602^2}{3} \times \frac{2}{3^2 \times 4} = \frac{2602^2}{9 \times 2} = 71\frac{1}{2}.$$

$$(10) \quad 100 \div 4\frac{1}{2} = 1\frac{1}{2} \times \frac{2}{3} = 20\frac{1}{3}.$$

$$(11) \quad \frac{1}{2} \text{ of } 7\frac{1}{2} \div \frac{2}{3} = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}.$$

$$(12) \quad \frac{1}{2} \text{ of } 50 \div 4\frac{1}{2} = \frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{2} = \frac{5 \times 25 \times 1}{13} = 1\frac{1}{13} = 9\frac{1}{13}.$$

$$(13) \quad 1\frac{1}{2} = \frac{3}{2}, \text{ and } 2\frac{1}{2} = \frac{5}{2}, \therefore \frac{3}{2} \div \frac{5}{2} = \frac{3}{2} \times \frac{2}{5} = \frac{3}{5}.$$

$$(14) \quad \frac{1}{2} \text{ of } 3\frac{1}{2} = \frac{1}{2} \text{ of } 1\frac{1}{2} = \frac{1}{2}, \text{ and } \frac{1}{2} \text{ of } 4\frac{1}{2} = \frac{1}{2} \text{ of } \frac{9}{2} = \frac{9}{4};$$

hence $\frac{1}{2} \div \frac{9}{4} = \frac{1}{2} \times \frac{4}{9} = \frac{2}{9} = 1\frac{2}{9}.$

$$(15) \quad 4\frac{1}{2} \div 5 = \frac{9}{2} \div \frac{5}{1} = \frac{9}{2} \times \frac{1}{5} = \frac{9}{10}.$$

$$(16) \quad 7\frac{1}{2} \div 9\frac{1}{2} = \frac{15}{2} \div \frac{19}{2} = \frac{15}{2} \times \frac{2}{19} = \frac{15 \times 3}{59} = \frac{45}{59}.$$

$$(17) \quad 20\frac{1}{2} \div 22\frac{1}{2} = 1\frac{1}{2} \div 1\frac{1}{2} = \frac{181 \times 12}{9 \times 265} = \frac{181 \times 4}{3 \times 265} = 7\frac{1}{2}.$$

$$(18) \quad \frac{1}{2} \text{ of } 7\frac{1}{2} \div 14 = \frac{1}{2} \text{ of } \frac{15}{2} \times \frac{1}{14} = \frac{15}{28} \times \frac{1}{14} = \frac{15}{392}.$$

RULE OF THREE IN FRACTIONS, p. 83.

$$(2) \quad \frac{1}{2} \text{ yd.} : \frac{2}{3} \text{ yds.} :: \frac{1}{2} \text{ l.} : \frac{2}{3} \times \frac{1}{2} \times \frac{2}{3} \text{ l.} = 4\text{ l. } 4\text{ s. } 5\frac{1}{2} \text{ d. } \frac{1}{2}$$

$$(6) \quad 1\frac{1}{2} \text{ of } 5 \text{ qrs.} : 2\frac{1}{2} \text{ of } 3 \text{ qrs.} :: \frac{1}{2} \text{ gui.};$$

or $\frac{4}{5} : \frac{7}{5} :: \frac{1}{2} \times \frac{3}{5} \text{ l.} : \frac{7}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{4}{5} =$

$$\frac{7 \times 7 \times 4}{2 \times 10 \times 9} = \frac{7 \times 7}{5 \times 9} = 4\frac{2}{9} \text{ l.} = 1\text{ l. } 1\text{ s. } 9\frac{1}{2} \text{ d. } \frac{1}{2}$$

$$(7) \quad \frac{1}{2} \text{ gall.} : \frac{1}{2} \times 4 \times 63 \text{ galls.} :: \frac{1}{2} \text{ l.} : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} =$$

$$5 \times 4 \times 7 = 140\text{ l.}$$

(8)

$$3\frac{1}{2} \times 24\frac{1}{2} = \frac{7}{8} \times \frac{7}{8} = \frac{49}{64} \text{ yds. ; and } 6s. 0\frac{1}{2}d. = 290 \text{ far. ;}$$

$$\therefore 1 \text{ yd. : } \frac{49}{64} \text{ yds. :: } \frac{49}{64} \text{ far. : } \frac{49}{64} \times \frac{49}{64} \text{ far.} = 24698\frac{1}{2} \text{ far.}$$

$$= 25l. 14s. 6\frac{1}{2}d. \frac{1}{4}.$$

(9)

$$7\frac{7}{8} \text{ in. : } 12 \text{ in. :: } 12 \text{ in. : } 12 \times 12 \times \frac{7}{8} = \frac{12 \times 6 \times 9}{35}$$

$$= 18\frac{1}{3} \text{ in. in length.}$$

(10)

The cost of carriage varies as the weight and distance jointly ;

$$\therefore 2\frac{1}{2} \text{ tons} \times 2\frac{7}{8} \text{ mi. : } \frac{1}{8} \text{ ton} \times 1 \text{ mi. :: } \frac{1}{8} \text{ gui. ;}$$

$$\text{or, } \frac{1}{8} \times \frac{7}{8} : \frac{1}{8} :: \frac{1}{8} \times 21 \times 12 \times 4 \text{ far. :}$$

$$\frac{3 \times 21 \times 12 \times 4 \times 2 \times 10}{40 \times 20 \times 5 \times 29} = \frac{3 \times 21 \times 12}{10 \times 5 \times 29} = \frac{3 \times 21 \times 6}{5 \times 5 \times 29} = 7\frac{1}{5} \text{ fa.}$$

(11)

$$8\frac{1}{2}s. : 5s. :: 6\frac{1}{8} \text{ oz. : } \frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} = \frac{1}{8} \text{ oz.} = 4\frac{1}{8} \text{ oz.}$$

(12)

$$11\frac{1}{8} \text{ hrs. : } 13\frac{1}{8} \text{ hrs. :: } 35\frac{1}{2} \text{ days : } \frac{7}{8} \times \frac{1}{8} \times \frac{1}{8} =$$

$$\frac{71 \times 109 \times 5}{8 \times 119} = 40\frac{1}{8} \frac{1}{8} \text{ days.}$$

(13)

$$5 \text{ qrs. : } 3 \text{ qrs. :: } 18\frac{7}{8} \text{ yds. : } \frac{1}{8} \times \frac{1}{8} = 11 \text{ yds. } 1 \text{ qr. } 1\frac{1}{2} \text{ nl.}$$

(14)

$$\frac{7}{8} \text{ yd. : } 1\frac{1}{8} \text{ yd. :: } 2\frac{1}{2} \text{ yds. : } \frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} = \frac{1}{8} \text{ yds. for 1 coat ;}$$

$$\therefore 976 \times \frac{1}{8} = \frac{488 \times 65}{7} = 4531 \text{ yds.} = 4531 \text{ yds. } 1 \text{ qr. } 2\frac{1}{2} \text{ nl.}$$

(15)

$$\frac{1}{8} \text{ yd. : } 1\frac{1}{8} \text{ yd. :: } 3\frac{1}{8} \text{ yds. : } \frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} = 3 \times 3 = 9 \text{ yds.}$$

DECIMAL FRACTIONS.

ADDITION.

(2)	(3)	(4)
376·25	3·5	276·
86·125	47·25	54·321
637·4725	927·01	·65
6·5	2·0073	112·
358·865	1·5	1·25
41·02	Sum = <u>981·2673</u>	·0463
Sum = <u>1506·2325</u>		Sum = <u>444·2673</u>

SUBTRACTION.

(2)	(3)
127·62	6213·725
13·725	162·25
Diff. <u>113·895</u>	Diff. <u>6051·475</u>
(4)	(5)
3760·279	1828·
423·0076	2·943764
Diff. <u>3337·2714</u>	Diff. <u>1825·056236</u>

MULTIPLICATION.

RULE I.

(2)	(3)
79·347	·63478
23·15	·8204
396735	253912
79347	126956
238041	507824
158694	·520773512 Prod.
<u>1836·88305</u> Prod.	

$$\begin{array}{r}
 (4) \\
 .385746 \\
 .00464 \\
 \hline
 1542984 \\
 2314476 \\
 1542984 \\
 \hline
 .00178986144 \text{ Prod.}
 \end{array}$$

$$\begin{array}{r}
 (5) \\
 1.570796 \\
 26.3719 \\
 \hline
 14137164 \\
 1570796 \\
 10995572 \\
 4712388 \\
 9424776 \\
 3141592 \\
 \hline
 41.4248750324 \text{ Prod.}
 \end{array}$$

RULE II.

$$\begin{array}{r}
 (2) \\
 245.378263 \\
 5834.27 \\
 \hline
 1717647841 \\
 49075653 \\
 9815130 \\
 736135 \\
 196302 \\
 12269 \\
 \hline
 17774.83330
 \end{array}$$

$$\begin{array}{r}
 (3) \\
 .248264 \\
 432527 \\
 \hline
 1737848 \\
 49653 \\
 12413 \\
 496 \\
 74 \\
 10 \\
 \hline
 .180049
 \end{array}$$

$$\begin{array}{r}
 (3) \\
 .248264 \\
 432527 \\
 \hline
 173785 \\
 4965 \\
 1241 \\
 50 \\
 7 \\
 1 \\
 \hline
 .18005
 \end{array}$$

$$\begin{array}{r}
 (3) \\
 .248264 \\
 432527 \\
 \hline
 17378 \\
 496 \\
 124 \\
 5 \\
 1 \\
 \hline
 .1800
 \end{array}$$

$$\begin{array}{r}
 (4) \\
 8634.875 \\
 7257.348 \\
 \hline
 6907900 \\
 345395 \\
 25905 \\
 6044 \\
 432 \\
 17 \\
 6
 \end{array}$$

$$\text{Product} = \underline{\underline{7285699}}$$

DIVISION OF DECIMALS.

(2)	(3)
$ \begin{array}{r} .7854)14.0000000(17.825, \&c. \\ \underline{7854} \\ 61460 \\ \underline{54978} \\ 64820 \\ \underline{62832} \\ 19880 \\ \underline{15708} \\ 41720 \\ \underline{39270} \\ 2450 \end{array} $	$ \begin{array}{r} 64.25)234.70525(3.653 \\ \underline{19275} \\ 41955 \\ \underline{38550} \\ 34052 \\ \underline{32125} \\ 19275 \\ \underline{19275} \end{array} $

(4)

$$\begin{array}{r}
 100)217.568(2.175, \&c. \\
 \underline{200} \\
 175 \\
 \underline{100} \\
 756 \\
 \underline{700} \\
 .568 \\
 \underline{500} \\
 68
 \end{array}$$

Obs. The quot. is the same as the dividend, only the decimal point is removed *two* places to the left hand.

(5)	(6)
$ \begin{array}{r} .7854)15.0000000(19.0985, \&c. \\ \underline{7854} \\ 71460 \\ \underline{70686} \\ 77400 \\ \underline{70686} \\ 67140 \\ \underline{62832} \\ 43080 \\ \underline{39270} \\ 3810 \end{array} $	$ \begin{array}{r} .1107)298.8900(2700 \\ \underline{2214} \\ 7749 \\ \underline{7749} \\ 00 \end{array} $

(7)
$ \begin{array}{r} .888)888.000(1000 \\ \underline{888} \\ 000 \end{array} $

CASE II.

$ \begin{array}{r} (2) \\ 2 \cdot 257432) 721 \cdot 17562 (319 \cdot 467, \&c. \text{ Ans.} \\ \underline{677230} \\ 43945 \\ \underline{22574} \\ 21371 \\ \underline{20317} \\ 1054 \\ \underline{903} \\ 151 \\ \underline{135} \\ 16 \\ \underline{15} \\ 1 \\ \hline \end{array} $	$ \begin{array}{r} (3) \\ 3 \cdot 14159) 12 \cdot 169825 (3 \cdot 87377 \text{ Ans.} \\ \underline{942477} \\ 274505 \\ \underline{251327} \\ 23178 \\ \underline{21991} \\ 1187 \\ \underline{942} \\ 245 \\ \underline{220} \\ 25 \\ \underline{22} \\ 3 \\ \hline \end{array} $
---	--

REDUCTION OF DECIMALS.

CASE I.

$ \begin{array}{r} (3) \\ 28) 5 \cdot 00000 (17857, \&c. \\ \underline{28} \\ 220 \\ \underline{196} \\ 240 \\ \underline{224} \\ 160 \\ \underline{140} \\ 200 \\ \underline{196} \\ 4 \\ \hline \end{array} $	$ \begin{array}{r} (4) \\ 182) 110 \cdot 00000 (6043956, \&c. \\ \underline{1092} \\ 800 \\ \underline{728} \\ 720 \\ \underline{546} \\ 1740 \\ \underline{1638} \\ 1020 \\ \underline{910} \\ 1100 \\ \underline{1092} \\ 8 \\ \hline \end{array} $
---	---

(5)	(6)
192)3·000000(015625	3842)275·000000(071577, &c.
192	26894
1080	6060
960	3842
1200	22180
1152	19210
480	29700
384	26894
960	28060
960	26894
	1166

(7)	(8)
1728 $\left\{ \begin{array}{l} 12 \\ 12 \\ 12 \end{array} \right \begin{array}{l} 13·0000000 \\ 1·0833333 \\ ·0902777 \\ ·0075231 \end{array}$	$2\frac{1}{4} \div 5\frac{1}{2} = \frac{1}{2} \times \frac{4}{11} = \frac{4}{3 \times 3} = \frac{4}{9}$
Ans.	=·444444.

CASE II., p. 91.

(2)	(3)	(4)
4 2·000 qrs.	4 2·000 nls.	60 50·400 sec.
20 4·500 cwt.	4 3·500 qrs.	60 15·840 min.
·225 ton.	·875 yd.	24 18·264 hrs.
		·761 day.

(5)	(6)	(7)
24 12·0 hrs.	4 3·0000000 f.	12 6·00000000 in.
365 109·5 days.	12 11·7500000 d.	3 1·50000000 ft.
·3 year.	27 $\left\{ \begin{array}{l} 3 \\ 9 \end{array} \right \begin{array}{l} 19·9791666s. \\ 6·6597222 \end{array}$	1760 17·50000000 yd.
	·7399691 moi.	·00994318 mi.

(8)	(9)
3 1·000000 b. c.	60 56·0000000 sec.
12 3·333333 in.	60 59·9333333 min.
3 0·277777 ft.	24 $\left\{ \begin{array}{l} 4 \\ 6 \end{array} \right \begin{array}{l} 23·9988888 hrs. \\ 5·9997222 \end{array}$
5 $\frac{1}{2}$ 1·092592 yd.	7 4·9999537 days.
2	4 1·7142791 week.
11 2·185184	·4285698 month.
40 0·198653 po.	
8 0·079716 fur.	
3 2·009964 mi.	
·669988 league.	

CASE III.

$$\begin{array}{r}
 (2) \\
 \cdot 76498 \text{ gui.} \\
 \underline{21} \\
 76498 \\
 152996 \\
 \underline{16\cdot 06458s.} \\
 12 \\
 \cdot 77496d. \\
 \underline{4} \\
 3\cdot 09984
 \end{array}$$

Ans. 16s. 0 $\frac{1}{2}$ d.

$$\begin{array}{r}
 (5) \\
 \cdot 6725 \text{ cwt.} \\
 \underline{4} \\
 2\cdot 6900 \text{ qr.} \\
 \underline{28} \\
 55200 \\
 13800 \\
 \underline{19\cdot 3200 \text{ lb.}} \\
 16 \\
 5\cdot 1200 \text{ oz.}
 \end{array}$$

Ans. 2 qrs. 19 lb. 5 oz.

$$\begin{array}{r}
 (3) \\
 \cdot 625s. \\
 \underline{12} \\
 7\cdot 500d. \\
 \underline{4} \\
 2\cdot 000 \\
 \underline{Ans. 7\frac{1}{2}d.}
 \end{array}$$

$$\begin{array}{r}
 (6) \\
 \cdot 67 \text{ lea.} \\
 \underline{3} \\
 2\cdot 01 \text{ mi.} \\
 \underline{8} \\
 0\cdot 08 \text{ fur.} \\
 \underline{40} \\
 3\cdot 20 \text{ po.} \\
 \underline{5\frac{1}{2}} \\
 1\cdot 10 \text{ yd.} \\
 \underline{3} \\
 0\cdot 30 \text{ ft.} \\
 \underline{12} \\
 3\cdot 60 \text{ in.} \\
 \underline{3} \\
 1\cdot 80 \text{ b. c.}
 \end{array}$$

Ans. 2 mi. 0 fur. 3 po. 1 yd. 3 in. 1 bar.

$$\begin{array}{r}
 (8) \\
 \cdot 461 \text{ ch.} \\
 \underline{36} \\
 2766 \\
 1383 \\
 \underline{16\cdot 596 \text{ bush.}} \\
 4 \\
 2\cdot 384 \text{ pks.}
 \end{array}$$

$$\begin{array}{r}
 (4) \\
 \cdot 8322916l. \\
 \underline{20} \\
 16\cdot 6458320s. \\
 \underline{12} \\
 7\cdot 7499840d. \\
 \underline{4} \\
 2\cdot 9999360f. \\
 \underline{Ans. 16s. 7\frac{1}{2}d.}
 \end{array}$$

$$\begin{array}{r}
 (7) \\
 \cdot 61 \text{ of a tun of wine.} \\
 \underline{4} \\
 2\cdot 44 \text{ hhd.} \\
 \underline{63} \\
 132 \\
 264 \\
 \underline{27\cdot 72 \text{ gall.}} \\
 4 \\
 2\cdot 88 \text{ qt.} \\
 \underline{2} \\
 1\cdot 76 \text{ pt.} \\
 \underline{Ans. 2 hhd. 27 galls.} \\
 [2 \text{ qts. 1 pt.}]
 \end{array}$$

Ans. 2 hhd. 27 galls.
[2 qts. 1 pt.]

Ans. 16 bush. 2 pks.

RULE OF THREE IN DECIMALS.

$ \begin{array}{r} (9) \\ .42857 \text{ mo.} \\ \underline{4} \\ 1.71428 \text{ wk.} \\ \underline{7} \\ 4.99996 \text{ days.} \\ \underline{24} \\ 399984 \\ 199992 \\ 23.99904 \text{ hr.} \\ \underline{60} \\ 59.94240 \text{ min.} \\ \underline{60} \\ 56.54400 \text{ sec.} \\ \hline \text{Ans. 1 wk. 4 days, 23 hrs. 59} \\ \quad \quad \quad \text{[min. 56 sec.]} \end{array} $	$ \begin{array}{r} (10) \\ 4 \overline{) 2.0 \text{ rd.}} \\ \underline{3.5 \text{ ac.}} \\ .092 \\ \underline{70} \\ 315 \\ \underline{3220 \text{ ac.}} \\ 4 \\ 1.2880 \text{ rd.} \\ \underline{40} \\ 11.5200 \text{ po.} \\ \hline \text{[11.52 p.} \\ \text{Ans. 1 rd.} \end{array} $
	$ \begin{array}{r} (11) \\ .9946 \text{ gui.} \times 21 = 20.8866s. \\ .3492l. \times 20 = 6.9840s. \\ \hline \text{Diff.} = 13.9026s. \\ \underline{12} \\ 10.8312d. \\ \underline{4} \\ 3.3248f. \\ \hline \text{Ans. 13s. } 10\frac{1}{4}d. \end{array} $

RULE OF THREE IN DECIMALS.

(2)

As 1 oz. : 1 lb. 10 oz. 10 dwts. 4 grs. :: 5s. 6d.
 or, 1 oz. : 22.50833 oz. :: 5.5s. ;
 $\therefore \text{price} = 22.50833 \times 5.5 = 123.7958s. = 6l. 3s. 9\frac{1}{4}d.$

(3)

As 10.5l. : 283.875l. :: 56 qrs.
 $\therefore \text{quantity} = \frac{56 \times 283.875}{10.5} = 1514\frac{2}{3} \text{ qrs.} = 504 \text{ Fl. ells, } 2\frac{2}{3} \text{ qr.}$

(4)

As 7s. 9 $\frac{1}{4}d.$: 25l. 18s. 1 $\frac{1}{4}d.$:: 4 qrs.
 or, .389583l. : 25.9072916l. :: 4 qrs.
 $\therefore \text{quantity} = \frac{4 \times 25.9072916}{.389583} = 266 \text{ qrs.} = 53 \text{ ells, } 1 \text{ qr.}$

(5)

As 14 lb. : 3 cwt. 3 qrs. 19 lb. :: 1l. 2s. 6d.
 or, 14 lb. : 439 lb. :: 1.125l.
 $\therefore \text{value} = \frac{1.125 \times 439}{14} = \frac{493.875}{14} = 35.2767l. = 35l. 5s. 6\frac{1}{4}d.$

$$\begin{array}{l} \text{(6)} \\ \text{As } 14\frac{1}{2} \text{ cwt. : } 4\frac{1}{2} \text{ cwt. :: 50 miles;} \\ \text{or, } 14\cdot5 : 4\cdot5 :: 50 \text{ miles.} \end{array}$$

$$\therefore \text{ distance} = \frac{50 \times 4\cdot5}{14\cdot5} = 15\cdot5172 \text{ miles} = 15 \text{ mi. } 4 \text{ fur. } 5 \text{ po.}$$

$$\begin{array}{l} \text{(7)} \\ \text{As } 20\text{s. : } 12\cdot5\text{s. :: 240 coins.} \\ \therefore \text{ number} = \frac{12\cdot5 \times 240}{20} = 150 \text{ coins.} \end{array}$$

$$\begin{array}{l} \text{(8)} \\ \text{As } \left\{ \begin{array}{l} 3 \text{ sailors} \\ 9\frac{1}{4} \text{ months} \end{array} \right\} : \left\{ \begin{array}{l} 100 \text{ sailors} \\ 28\frac{1}{2} \text{ months} \end{array} \right\} :: 40\frac{1}{3}l. \\ \text{or, } \left\{ \begin{array}{l} 3 \text{ sailors} \\ 9\cdot25 \text{ months} \end{array} \right\} : \left\{ \begin{array}{l} 100 \text{ sailors} \\ 28\cdot428571 \text{ months} \end{array} \right\} :: 40\cdot2l. \\ \therefore \frac{2842\cdot8571 \times 40\cdot2}{27\cdot75} l. = 4118l. \text{ 6s. very nearly.} \end{array}$$

$$\begin{array}{l} \text{(9)} \\ \text{As } 1 \text{ grain : } 1\cdot5 \text{ oz. :: } 3\cdot25\text{s.} \\ \text{or, } 1 \text{ grain : } 720 \text{ grains :: } 3\cdot25\text{s.} \\ \therefore \text{ he sold the stone for } 3\cdot25 \times 720 \text{ shillings, or } 117l.; \\ \text{and his gain} = 117l. - 63l. = 54l. \\ \text{Also, to find the gain per cent., say as } 63l. : 54l. :: 100l. \\ \therefore \text{ gain per cent.} = \frac{54 \times 100}{63} l. = 85l. \text{ 14s. } 3\frac{1}{2}d. \end{array}$$

CIRCULATING DECIMALS, p. 95.

CASE I.

$$\text{(2)} \quad \dot{3} = \frac{3}{9} = \frac{1}{3}.$$

$$\text{(3)} \quad \dot{0}9 = \frac{9}{99} = \frac{1}{11}.$$

$$\text{(4)} \quad \dot{0}4 = \frac{4}{99}.$$

$$\text{(5)} \quad \dot{7}69230 = \frac{769230}{999999} = \frac{85470}{111111} = \frac{19}{13}.$$

$$\text{(6)} \quad \dot{1}42857 = \frac{142857}{999999} = \frac{1}{7}.$$

$$\text{(7)} \quad \dot{4}2\cdot6\dot{3} = \frac{426300}{9999} = \frac{142100}{3333}.$$

CASE II.

(4)

$$\cdot\dot{5}9\dot{2}\dot{5} = \frac{5925 - 5}{9990} = \frac{5920}{9990} = \frac{592}{999} = \frac{1}{2} \frac{1}{7} \text{ Ans.}$$

(5)

$$\cdot 0098\dot{7} = \frac{987 - 9}{99000} = \frac{978}{99000} = \frac{1}{10} \frac{1}{10} \frac{1}{10} \text{ Ans.}$$

(6)

$$4\cdot\dot{7}54\dot{3} = \frac{47543 - 47}{9990} = \frac{47496}{9990} = \frac{7}{10} \frac{1}{10} \frac{1}{10} \text{ Ans.}$$

(7)

$$\cdot 00849713\dot{3} = \frac{8497133 - 8}{999999000} = \frac{8497125}{999999000} = \frac{1}{125} \frac{1}{10} \frac{1}{10} \text{ Ans.}$$

ADDITION OF CIRCULATING DECIMALS.

(2)

$$\cdot\dot{3}\dot{6} = \frac{36}{99}; \cdot\dot{0}\dot{9} = \frac{9}{99} \therefore \text{the sum} = \frac{36 + 9}{99} = \frac{45}{99} = \cdot\dot{4}\dot{5} \text{ Ans.}$$

(3)

$$\cdot\dot{6} = \frac{6}{9}; \text{ and } \cdot\dot{1}4285\dot{7} = \frac{142857}{999999},$$

$$\therefore \text{the sum} = \frac{666666}{999999} + \frac{142857}{999999} = \frac{809523}{999999} = \cdot\dot{8}0952\dot{3} \text{ Ans.}$$

(4)

$$\cdot\dot{3} = \frac{3}{9}; \cdot\dot{9}4\dot{5} = \frac{945}{999}, \text{ and } \cdot\dot{7}6923\dot{0} = \frac{769230}{999999};$$

$$\therefore \text{the sum} = \frac{333333}{999999} + \frac{945945}{999999} + \frac{769230}{999999} = 2\cdot\dot{0}4850\dot{8} \text{ Ans.}$$

(5)

$$\cdot\dot{6}7\cdot\dot{3}4\dot{5} = \frac{67345 - 673}{990} = \frac{66672}{990}; \quad 8\cdot\dot{6}2\dot{1} = \frac{8621 - 8}{999} = \frac{8613}{999};$$

$$\cdot\dot{2}\dot{4} = \frac{24}{99}, \text{ and } \cdot\dot{8} = \frac{8}{9}; \text{ which fractions being reduced to a}$$

common denominator, the sum of the numerators divided
the denominator will give the sum = $77\cdot\dot{0}98389\dot{2}$.

$$\begin{aligned} & \text{(6)} \\ \cdot\dot{5} &= \frac{5}{9}; \cdot\dot{2}\dot{5} = \frac{25}{99}; \cdot\dot{4}\dot{7} = \frac{47-4}{90} = \frac{43}{90}; \cdot\dot{6}\dot{5}\dot{1} = \frac{651}{999}, \text{ and } \cdot\dot{3}\dot{4}\dot{5} \\ &= \frac{342}{990}. \end{aligned}$$

The fractions being reduced to a common denominator, their sum = $\frac{22829625}{9999990} = 2\cdot\dot{2}8\dot{2}964\dot{7}$; which, together with 93, the integral part, will give the sum required = $95\cdot\dot{2}8\dot{2}964\dot{7}$.

$$\begin{aligned} & \text{(7)} \\ \cdot\dot{8}\dot{1}\dot{4} &= \frac{814}{999}; \cdot\dot{5} = \frac{5}{10}; \cdot\dot{2}\dot{6} = \frac{26-2}{90} = \frac{24}{90}; \cdot\dot{8}\dot{3} = \frac{83-8}{90} = \frac{75}{90}; \\ \cdot\dot{0}\dot{9} &= \frac{9}{99}; \text{ the sum of which fractions, when reduced to a} \\ \text{common denominator} &= \frac{275354}{109890} = 2\cdot\dot{5}\dot{0}\dot{5}723\dot{9}; \text{ which, to-} \\ \text{gether with 221, the integral part, will give the required} \\ \text{sum} &= 223\cdot\dot{5}\dot{0}\dot{5}723\dot{9}. \end{aligned}$$

$$\begin{aligned} & \text{(8)} \\ \cdot\dot{3}\dot{5}\dot{7} &= \frac{357}{1000}; \cdot\dot{3}\dot{8} = \frac{38-3}{90} = \frac{35}{90}; \cdot\dot{2}\dot{1} = \frac{21-2}{90} = \frac{19}{90}; \cdot\dot{2}\dot{9}\dot{6}\dot{5} \\ &= \frac{2965-296}{9000} = \frac{2669}{9000}; \cdot\dot{8}\dot{4}\dot{9}\dot{6} = \frac{8496-849}{9000} = \frac{7647}{9000}; \cdot\dot{1}\dot{7}\dot{6} = \\ \frac{176-17}{900} &= \frac{159}{900}; \cdot\dot{5}\dot{2}\dot{3} = \frac{523-52}{900} = \frac{471}{900}; \cdot\dot{3}\dot{0}\dot{0}\dot{4}\dot{8} = \\ \frac{30048-3004}{90000} &= \frac{27044}{90000}; \text{ the sum of which fractions, when} \\ \text{reduced to a common denominator} &= \frac{279334}{90000} = 3\cdot\dot{1}\dot{0}\dot{3}\dot{7}\dot{1}; \\ \text{which, with 5971, the integral part, will give the sum} \\ &= 5974\cdot\dot{1}\dot{0}\dot{3}\dot{7}\dot{1}. \end{aligned}$$

$$\begin{aligned} & \text{(9)} \\ \cdot\dot{1}\dot{6}\dot{2} &= \frac{162}{999}; \cdot\dot{0}\dot{9} = \frac{9}{99}; \cdot\dot{9}\dot{3} = \frac{93}{99}; \cdot\dot{2}\dot{6} = \frac{26-2}{90} = \frac{24}{90}; \\ \cdot\dot{7}\dot{6}\dot{9}\dot{2}\dot{3}\dot{0} &= \frac{769230}{999999}; \cdot\dot{0}\dot{8}\dot{3} = \frac{83-8}{900} = \frac{75}{900}; \cdot\dot{5} = \frac{5}{10}; \cdot\dot{8}\dot{1}\dot{4} = \\ \frac{814}{999}; & \text{ the sum of which fractions, when reduced to a com-} \end{aligned}$$

mon denominator = $\frac{3989157865}{1099998900} = 3.62651077$; which, together with 336, the integral part, will give the required sum = 339.62651077 .

SUBTRACTION OF CIRCULATING DECIMALS.

(2)

$476.\dot{3}\dot{2} = 476\frac{32}{100} = 476\frac{8}{25} = 475\frac{100}{25} - \frac{12}{25}$
 and $84.769\dot{7} = 84\frac{7697}{10000}$; \therefore the diff. = $391\frac{100}{10000} = 391.5524$.

(3)

$3.856\dot{4} = 3\frac{8564}{10000}$; and $.38\dot{2} = \frac{382}{1000} = \frac{3820}{10000}$;
 whence $3\frac{8564}{10000} - \frac{3820}{10000} = \frac{38179 - 3784}{9900} = \frac{34395}{9900} = 3.474\dot{2}$.

(4)

$42\dot{7} = \frac{423}{990}$; and $.03\dot{4} = \frac{34}{990}$; the diff. = $\frac{389}{990} = .39\dot{2}$.

(5)

$3.85\dot{6} = 3\frac{856}{1000}$; $.038 = \frac{38}{1000}$;
 $\therefore 3\frac{856}{1000} - \frac{38}{1000} = 3\frac{817}{1000} = 3.817$ = difference.

(6)

$127.462\dot{7} = 127\frac{4627}{10000}$; and $48.\dot{6} = \frac{48600}{999}$;
 therefore $127\frac{4627}{10000} - \frac{48600}{999} = 78.814129$ Ans.

MULTIPLICATION OF CIRCULATING DECIMALS.

(2)

$37.2\dot{3} = 37\frac{23}{100} = 37\frac{7}{30}$; and $.2\dot{6} = \frac{24}{90} = \frac{8}{30}$;
 hence $\frac{1117}{30} \times \frac{8}{30} = \frac{8936}{900} = 9.92\dot{8}$.

(3)

$8574.\dot{3} = 8574\frac{1}{3} = \frac{25723}{3}$; and $87.\dot{5} = 87\frac{5}{9} = \frac{788}{9}$;
 $\therefore \frac{25723}{3} \times \frac{788}{9} = \frac{20269724}{27} = 750730.51\dot{8}$.

(4)

$$3\dot{9}7\dot{3} \times 8 = 3\frac{7}{11} \times 8 = \frac{3970 \times 8}{999} = \frac{31760}{999} = 31\dot{7}9\dot{1}.$$

(5)

$$49640\cdot54 \times \frac{70503}{99999} = \frac{3499806991\cdot62}{99999} = 34998\cdot419900\dot{3}.$$

(6)

$$7\dot{7}2 = 7\frac{2}{11} = 7\frac{8}{11}; \text{ and } \dot{2}9\dot{7} = \frac{297}{999} = \frac{11}{37};$$

$$\therefore \frac{85}{11} \times \frac{11}{37} = \frac{85}{37} = 2\dot{2}9\dot{7} \text{ Ans.}$$

DIVISION OF CIRCULATING DECIMALS.

(2)

$$319\cdot28007\dot{1}1\dot{2} = 319\cdot28007\frac{112}{99900000} = \frac{31896079105}{99900000}$$

$$\text{also } 764\dot{5} = 764\frac{5}{9} = \frac{6881}{9};$$

$$\therefore \frac{31896079105}{99900000} \div \frac{6881}{9} = \frac{31896079105}{76379100000} = 41\dot{7}6 \text{ Ans.}$$

(3)

$$234\dot{6} = 234\frac{2}{3} = 2\frac{2}{3}; \text{ and } \dot{7} = \frac{7}{9};$$

$$\therefore 2\frac{2}{3} \div \frac{7}{9} = 2\frac{2}{3} \times \frac{9}{7} = 2\frac{6}{7} = 301\dot{7}1428\dot{5}.$$

(4)

$13\cdot516953\dot{3} = 13\frac{169533}{1000000}$; also $4\dot{2}9\dot{7} = 4\frac{297}{1000}$; these two fractions reduced to a common denomination become

$\frac{135169398}{9999990}$ and $\frac{42972930}{9999990}$; hence the former divided by

the latter = $\frac{135169398}{42972930} = 3\cdot14\dot{5}$ the quotient required.

(5)

$41\cdot971\dot{7} = 41\frac{17}{144}$; and $34\cdot12 = 34\frac{1}{3}$; the fractions reduced to a common denominator become $\frac{4196760}{99990}$ and $\frac{3411881}{99990}$; the former divided by the latter = $\frac{4196760}{3411881} = 1\cdot23$.

DUODECIMALS.

(3)		(4)		(5)	
ft.	in.	ft.	in.	ft.	in.
14	9	7	5	7	10
4	6	3	9	8	11
<hr/>		<hr/>		<hr/>	
59	0	22	3	62	8
7	4	5	6	7	2
6		9		2	
<hr/>		<hr/>		<hr/>	
Ans. 66	4 6	Ans. 27	9 9	Ans. 69	10 2

(6)		(7)	
ft.	in.	ft.	in.
5	7	24	6
1	10	16	3
<hr/>		<hr/>	
5	7	392	0
4	7 10	6	1 6
<hr/>		<hr/>	
10	2 10	9)398	1 6 sq. feet.
	6	44	2 10 sq. yards.
20)61	5 0		3
<hr/>		<hr/>	
£3	1 5 Ans.	2,0)13,2	8 6
		£6	12 8½* Ans.

* It may perhaps require some explanation to enable the student to perceive how this result is obtained; but let him consider that the row 44 2 10 means 44 square yards, $\frac{2}{3}$ of a square yard, and $\frac{10}{144}$ of a square yard. And the line 132 8 6 is formed by multiplying 44 2 10 by 3, and carrying 1 for each 12; because they are duodecimals. Again, the line 132 8 6 means 132 shillings, $\frac{8}{12}$ of a shilling, and $\frac{6}{144}$ of a shilling; or 132 shillings, 8 pence, and 2 things.

$$\begin{array}{r}
 (8) \\
 \begin{array}{r}
 \text{ft. in.} \\
 7 \ 10 \\
 6 \ 8 \\
 5 \ 4 \\
 \hline
 19 \ 10 \\
 3 \ 11 \\
 \hline
 59 \ 6 \\
 18 \ 2 \ 2 \\
 \hline
 77 \ 8 \ 2 \\
 3 \\
 \hline
 233 \ 0 \ 6 \\
 2 \\
 \hline
 456 \ 1 \ 0 \\
 7 \\
 \hline
 12)3262 \ 7 \ 0 \\
 20)27,1 \ 10 \ 7 \\
 \hline
 \pounds 13 \ 11 \ 10\frac{1}{2} \text{ Ans.}
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 (9) \\
 \begin{array}{r}
 \text{ft. in.} \\
 74 \ 9 \\
 11 \ 6 \\
 \hline
 822 \ 3 \\
 37 \ 4 \ 6 \\
 \hline
 9)859 \ 7 \ 6 \\
 95 \ 6 \ 2 \\
 \hline
 3 \\
 286 \ 6 \ 6 \\
 6d. \text{ is } \frac{1}{2} \ 47 \ 9 \ 1 \\
 3d. \text{ is } \frac{1}{2} \ 23 \ 10 \ 6 \ 6 \\
 1\frac{1}{2}d. \text{ is } \frac{1}{2} \ 11 \ 11 \ 3 \ 3 \\
 \hline
 2,0)37,0 \ 1 \ 4 \ 9 \text{ [exact.} \\
 \pounds 18 \ 10 \ 1\frac{1}{2} + 1\frac{1}{4}d.
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 (10) \\
 \begin{array}{r}
 \text{ft. in.} \\
 58 \ 6 \\
 54 \ 9 \\
 \hline
 3159 \ 0 \\
 43 \ 10 \ 6 \\
 \hline
 9)3202 \ 10 \ 6 \\
 355 \ 10 \ 6 \\
 4 \\
 \hline
 1423 \ 6 \ 0 \\
 \frac{1}{2}d. \text{ is } \frac{1}{2} \ 177 \ 11 \ 3 \\
 \frac{1}{4}d. \text{ is } \frac{1}{4} \ 88 \ 11 \ 7 \ 6 \\
 \hline
 12)1690 \ 4 \ 10 \ 6 \\
 2,0)14,0 \ 10 \ 4 \ 10 \\
 \hline
 \pounds 7 \ 0 \ 10\frac{1}{2}d. + 1\frac{3}{4}d.
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 (11) \\
 \begin{array}{r}
 \text{ft. in.} \\
 97 \ 8 \\
 9 \ 10 \\
 \hline
 879 \ 0 \\
 81 \ 4 \ 8 \\
 \hline
 9)960 \ 4 \ 8 \text{ sq. feet.} \\
 106 \ 8 \ 6 \ 2 \ 8 \text{ sq. yds.} \\
 2 \\
 \hline
 213 \ 5 \ 0 \ 5 \ 4 \\
 6d. \text{ is } \frac{1}{2} \ 53 \ 4 \ 3 \ 1 \ 4 \\
 2d. \text{ is } \frac{1}{2} \ 17 \ 9 \ 5 \ 0 \ 5 \\
 \frac{1}{2}d. \text{ is } \frac{1}{2} \ 4 \ 5 \ 4 \ 3 \ 1 \\
 \frac{1}{4}d. \text{ is } \frac{1}{4} \ 2 \ 2 \ 8 \ 1 \ 6 \\
 \hline
 2,0)29,1 \ 2 \ 8 \ 11 \ 8 \\
 \pounds 14 \ 11 \ 2d. + \frac{8}{12}d. + 1\frac{1}{4}d. \\
 \hline
 \text{[} + 1\frac{1}{8}d.
 \end{array}
 \end{array}$$

<p>(12)</p> <table border="0"> <tr><td>ft.</td><td>in.</td></tr> <tr><td>8</td><td>3</td></tr> <tr><td>6</td><td>6</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td>49</td><td>6</td></tr> <tr><td>4</td><td>1 6</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td>9</td><td>53 7 6</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td></td><td>5 11 6 sq. yards.</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td></td><td>6</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td></td><td>35 9 0</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td>6d. is $\frac{1}{2}$</td><td>2 11 9</td></tr> <tr><td>1$\frac{1}{2}$d. is $\frac{1}{4}$</td><td>0 8 11 3</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td>2,0</td><td>3,9 5 8 3 [$\frac{3}{4}$d.</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td></td><td>£1 19s. 5d. + $\frac{3}{4}$d. +</td></tr> </table>	ft.	in.	8	3	6	6	<hr/>		49	6	4	1 6	<hr/>		9	53 7 6	<hr/>			5 11 6 sq. yards.	<hr/>			6	<hr/>			35 9 0	<hr/>		6d. is $\frac{1}{2}$	2 11 9	1 $\frac{1}{2}$ d. is $\frac{1}{4}$	0 8 11 3	<hr/>		2,0	3,9 5 8 3 [$\frac{3}{4}$ d.	<hr/>			£1 19s. 5d. + $\frac{3}{4}$ d. +	<p>(13)</p> <table border="0"> <tr><td>ft.</td><td>in.</td></tr> <tr><td>30</td><td>6</td></tr> <tr><td></td><td>3</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td>4</td><td>91 6</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td></td><td>22 10</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td></td><td>52 8</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td></td><td>1189 6 0 [rafter.</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td></td><td>15 3</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td></td><td>1204 9 = length of</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td></td><td>2</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td>100</td><td>2409 6 = length of</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td></td><td>24 1 1 8, &c.</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td></td><td>10</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td></td><td>240 11 4 8</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td>6 is $\frac{1}{2}$</td><td>12 0 6 10</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td>2,0</td><td>25,2 11 11 6 [roof.</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td></td><td>£12 12s. 11d. + $\frac{1}{4}$d. +</td></tr> </table>	ft.	in.	30	6		3	<hr/>		4	91 6	<hr/>			22 10	<hr/>			52 8	<hr/>			1189 6 0 [rafter.	<hr/>			15 3	<hr/>			1204 9 = length of	<hr/>			2	<hr/>		100	2409 6 = length of	<hr/>			24 1 1 8, &c.	<hr/>			10	<hr/>			240 11 4 8	<hr/>		6 is $\frac{1}{2}$	12 0 6 10	<hr/>		2,0	25,2 11 11 6 [roof.	<hr/>			£12 12s. 11d. + $\frac{1}{4}$ d. +
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INVOLUTION.

(4)
 $35 \times 35 = 1225$ = square of 35;
 $1225 \times 35 = 42875$ = cube of 35.

(5)
 $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{81}{256} = (\frac{3}{4})^4$ Ans.

(6)
 $1\frac{1}{2} = \frac{3}{2}$; hence $\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = \frac{27}{8} = 2\frac{3}{8}$ Ans.

(7)
 $6 \cdot 03 \times 6 \cdot 03 \times 6 \cdot 03 = 219 \cdot 256227 = (6 \cdot 03)^3$; and
 $(6 \cdot 03)^3 \times (6 \cdot 03)^3 = 48073 \cdot 293078275529$.

(8)
 $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{64} = (\frac{1}{2})^6$, the sixth power of $\frac{1}{2}$.

(9)
 $\cdot 009 \times \cdot 009 \times \cdot 009 = \cdot 000000729$.

(10)
 $3\frac{1}{2} = \frac{7}{2}$; and $\frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} = \frac{2401}{16} = 133\frac{13}{16}$.

EVOLUTION.

EXTRACTION OF SQUARE ROOT.

(3)

$$\begin{array}{r}
 106929(327 \text{ Ans.} \\
 \underline{9} \\
 62)169 \\
 \underline{124} \\
 647)4529 \\
 \underline{4529}
 \end{array}$$

(4)

$$\begin{array}{r}
 16777216(4096 \text{ Ans.} \\
 \underline{16} \\
 809)7772 \\
 \underline{7281} \\
 8186)49116 \\
 \underline{49116}
 \end{array}$$

(5)

$$\begin{array}{r}
 43046721(6561 \text{ Ans.} \\
 \underline{36} \\
 125)704 \\
 \underline{625} \\
 1306)7967 \\
 \underline{7836} \\
 13121)13121 \\
 \underline{13121}
 \end{array}$$

(6)

$$\begin{array}{r}
 3.17218120(1.78106, \&c. \\
 \underline{1} \\
 27)217 \\
 \underline{189} \\
 348)2821 \\
 \underline{2784} \\
 3561)3781 \\
 \underline{3561} \\
 356206)2202000 \\
 \underline{2137236} \\
 64764, \&c.
 \end{array}$$

(7)

$$\begin{array}{r}
 .0003275400(.01809, \&c. \\
 \underline{1} \\
 28)227 \\
 \underline{224} \\
 3609)35400 \\
 \underline{32481} \\
 2919, \&c.
 \end{array}$$

(8)

$$\begin{array}{r}
 \sqrt[4]{\frac{1}{8}} = .41666666(.6454, \&c. \\
 \underline{36} \\
 124)566 \\
 \underline{496} \\
 1285)7066 \\
 \underline{6425} \\
 12904)64166 \\
 \underline{51616} \\
 12550, \&c.
 \end{array}$$

(9)

$$6\frac{1}{4} = 6.400000(2.529, \&c.$$

$$\begin{array}{r} 4 \\ 45 \overline{)240} \\ \underline{225} \\ 502 \overline{)1500} \\ \underline{1004} \\ 5049 \overline{)49600} \\ \underline{45441} \\ \underline{4159}, \&c. \end{array}$$

(10)

$$10.00000000(3.1622, \&c.$$

$$\begin{array}{r} 9 \\ 61 \overline{)100} \\ \underline{61} \\ 626 \overline{)3900} \\ \underline{3756} \\ 6322 \overline{)14400} \\ \underline{12644} \\ 63242 \overline{)175600} \\ \underline{126484} \\ \underline{49116}, \&c. \end{array}$$

(11)

$$2.00000000(1.4142135$$

$$\begin{array}{r} 1 \\ 24 \overline{)100} \\ \underline{96} \\ 281 \overline{)400} \\ \underline{281} \\ 2824 \overline{)11900} \\ \underline{11296} \\ 28282 \overline{)60400} \\ \underline{56564} \\ \underline{3836}, \&c. \end{array}$$

(12)

$$5.00000000(2.23606$$

$$\begin{array}{r} 4 \\ 42 \overline{)100} \\ \underline{84} \\ 443 \overline{)1600} \\ \underline{1329} \\ 4466 \overline{)27100} \\ \underline{26796} \\ 447206 \overline{)3040000} \\ \underline{2683236} \\ \underline{356764} \end{array}$$

EXTRACTION OF THE CUBE ROOT.

RULE I.

(2)	(3)
$ \begin{array}{r} 7 \dots 49 \dots 343 \\ \underline{7} \\ 14 \dots 98 \\ \underline{7} \\ 213 \dots 639 \\ \underline{15339} \dots 46017 \end{array} $	$ \begin{array}{r} 1 \dots 1 \dots 1 \\ \underline{1} \\ 2 \dots 2 \\ \underline{1} \\ 303 \dots 909 \\ \underline{30909} \dots 92727 \end{array} $

(4)

$ \begin{array}{r} 6 \dots 36 \dots 216 \\ \underline{6} \\ 12 \dots 72 \\ \underline{6} \\ 187 \dots 1309 \\ \underline{7} \\ 194 \dots 1358 \\ \underline{7} \\ 2018 \dots 16146 \\ \underline{8} \\ 2026 \dots 16208 \\ \underline{8} \\ ,20,34 \\ \underline{137905} 4 \\ \underline{20} 3 \\ \underline{137925} 7 \dots 137926 \\ \underline{20} 3 \\ \underline{137946} \\ \underline{1} 2 \\ \underline{137958} \\ \underline{1} 2 \\ \underline{137,9,7} \end{array} $	$ \begin{array}{r} 311897 \cdot 910 (67 \cdot 81683 \\ \underline{216} \\ 95897 \\ \\ 84763 \\ \underline{11134910} \\ \\ 10902768 \\ \underline{232142} \\ \\ 82775 \\ \underline{11441} \\ 11038 \\ \underline{403} \\ \underline{413} \end{array} $
---	--

(5)	(6)
$ \begin{array}{r} 7 \dots 49 \dots 343 \\ \hline 7 \quad \quad \quad 62224 \\ 14 \dots 98 \\ \hline 7 \quad 147 \\ 214 \dots 856 \\ \hline 15556 \dots 62224 \end{array} $	$ \begin{array}{r} 7 \dots 49 \dots 343 \\ \hline 7 \quad \quad \quad 101194 \\ 14 \dots 98 \\ \hline 7 \quad 147 \\ 216 \dots 1296 \\ \hline 6 \quad 15996 \dots 95976 \\ 222 \dots 1332 \\ \hline 6 \quad 17328 \\ 2283 \dots 6849 \\ \hline 1739649 \dots 5218947 \end{array} $

(7)

$$\begin{array}{r}
 2 \dots \dots 4 \dots \dots \dots 8 \\
 \hline
 2 \quad \quad \quad 5666 \\
 4 \dots \dots 8 \\
 \hline
 2 \quad \quad 12 \\
 63 \dots \dots 189 \\
 \hline
 3 \quad \quad 1389 \dots \dots 4167 \\
 66 \dots \dots 198 \\
 \hline
 3 \quad \quad 1587 \\
 699 \dots \dots 6291 \\
 \hline
 9 \quad \quad 164991 \dots \dots 1484919 \\
 708 \dots \dots 6372 \\
 \hline
 9 \quad \quad 17136300 \\
 71708 \quad \quad 573664 \\
 \hline
 1714203664 \dots \dots 13713629312 \\
 \hline
 1034037354
 \end{array}$$

(8)

$$\frac{1520}{5130} = \frac{152}{513} = \frac{8}{27} \text{ and } \sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}.$$

(9)

$$\begin{array}{r}
 \begin{array}{r}
 8 \dots\dots 64 \dots\dots 512 \\
 \hline
 8 \\
 16 \dots\dots 128 \\
 \hline
 8 \quad 192 \\
 247 \dots\dots 1729 \\
 \hline
 7 \quad 20929 \dots\dots 146503 \\
 254 \dots\dots 1778 \\
 \hline
 7 \quad 22707 \\
 2613 \quad 7839 \\
 \hline
 3 \quad 2278539 \dots\dots 6835617 \\
 2616 \dots\dots 7848 \\
 \hline
 3 \quad 228638 \overline{)7} \\
 ,26,19 \quad 130 \overline{)9} \\
 \hline
 228769 \overline{)6} \dots\dots 1143848 \\
 130 \overline{)9} \quad 184201 \\
 \hline
 22890 \overline{)0} \\
 2 \overline{)0} \\
 \hline
 22892 \dots\dots 183136 \\
 2 \quad 1065 \\
 \hline
 228,9,4 \quad 915
 \end{array}
 \end{array}$$

(10)

$$\begin{array}{r}
 \begin{array}{r}
 7 \dots\dots 49 \dots\dots 343 \\
 \hline
 7 \quad 131552 \quad \therefore 78 \times 78 = 6084 \text{ sq. in.} \\
 14 \dots\dots 98 \\
 \hline
 7 \quad 147 \\
 218 \dots\dots 1744 \\
 \hline
 16444 \dots\dots 131552
 \end{array}
 \end{array}$$

RULE II.

(2)

In this example $4\cdot913 = (1\cdot7)^3 =$ that rational cube number which is nearest to 5: hence let 1·7 be the assumed root: then by the rule

$$2 \times (4\cdot913) + 5 : 2 \times 5 + 4\cdot913 :: 1\cdot7;$$

$$\text{or, } 14\cdot826 : 14\cdot913 :: 1\cdot7$$

$$\begin{array}{r} 1\cdot7 \\ 14\cdot826 \overline{) 25\cdot3521} (1\cdot709976 \text{ Ans.} \end{array}$$

(3)

Here 64 is the nearest cube to 81.

$$\therefore 2 \times 64 + 81 : 2 \times 81 + 64 :: 4;$$

$$\text{or, } 209 : 226 :: 4 : 4\cdot3 = \text{first approximation.}$$

$$\text{Also, } (4\cdot3)^3 = 79\cdot507;$$

$$\text{or, } 240\cdot014 : 241\cdot507 :: 4\cdot3 : 4\cdot3267 \text{ the 2nd approxim.}$$

(4)

Now $\frac{1}{4} = \cdot4444$, &c., and assume $\cdot7$ as the cube root.

$$\text{Then as } 2 \times (\cdot7)^3 + \cdot4444, \&c. : \cdot8888, \&c. + (\cdot7)^3 :: \cdot7;$$

$$\text{or, as } 1\cdot13044, \&c. : 1\cdot23188, \&c. :: \cdot7$$

$$\begin{array}{r} \cdot7 \\ 1\cdot13044 \overline{) 862316} (\cdot763, \&c. \\ 791308 \\ \hline 71008 \end{array}$$

(5)

In this example 4·8 is a near approximation to the cube root of 117; for $(4\cdot8)^3 = 110\cdot592$.

$$\text{Then as } 2(110\cdot592) + 117 : 2 \times 117 + 110\cdot592 :: 4\cdot8;$$

$$\text{or, } 338\cdot184 : 344\cdot592 :: 4\cdot8 : 4\cdot89095 \text{ Ans.}$$

(6)

The cube which is nearest to 51·125 is $(3\cdot7)^3 = 50\cdot653$; hence

$$2 \times (50\cdot653) + 51\cdot125 : 2 \times (51\cdot125) + 50\cdot653 :: 3\cdot7;$$

$$\text{or, } 152\cdot431 : 152\cdot903 :: 3\cdot7 : 3\cdot711457 \text{ Ans.}$$

(7)

$\frac{1}{2} = \cdot5$; and $(\cdot7)^3 = \cdot343$, which may be taken as the first approximate value.

$$\text{Hence } \cdot686 + \cdot5 : 2 \times \cdot5 + \cdot343 :: \cdot7;$$

$$\text{or, } 1\cdot186 : 1\cdot343 :: \cdot7 : \cdot79266 \text{ a 2nd approx. value.}$$

If this value be used, we shall obtain at the next process 937005, &c., for the cube root of $\frac{1}{2}$.

TO EXTRACT THE ROOTS OF POWERS IN GENERAL.

(2)

Here $n=4$, $r=1$, $A=1$, $N=2$; hence by the rule

$$(n+1) \cdot A + (n-1) \cdot N : (n+1) N + (n-1) A :: r;$$

i. e., $11 : 13 :: 1 : 1.18$ the new root.

Substitute the value of the root, and there comes out the following proportion:

$$5 \times (1.18)^4 + 6 : 10 + 3 \times (1.18)^4 :: 1.18;$$

$$\text{or, } 15.6938888 : 15.8163328 :: 1.18$$

1.18

$$15.6938888 \overline{) 15.8163328}$$

1.18920, &c. Ans.

In cases where the 4th root or 8th root is required, it may be found, by first extracting the square root, and then the square root of this square root will be the 4th root; and the square root of this last will be the 8th root; or as in the following example.

(3)

The 8th root of 2 is required.

Now the 4th root of $2 = 1.18920$, &c., as in Ex. 2, just given.

$1.18920(1.0905$, &c., the 8th root required.

1

$$209 \overline{) 1892}$$

1881

$$21805 \overline{) 110000}$$

109025

975, &c.

(4)

The first root to be assumed is evidently 1, and $n=7$; $N=2$; $A=1$; $r=1$.

$$\text{Hence } \left\{ \begin{matrix} (n+1) A = 8 \\ (n-1) N = 12 \end{matrix} \right\} \text{ also } \left\{ \begin{matrix} (n+1) N = 16 \\ (n-1) A = 6 \end{matrix} \right\}$$

$\therefore 20 : 22 :: 1 : \frac{1}{10}$ = the first approximation.

In the next process, $N=2$, $n=7$, $r=\frac{1}{10}$, and $A=(\frac{1}{10})^7$
 $= (1.1)^7 = 1.9487171$.

$$\begin{array}{rcl} \therefore (n+1)A = 15.5897368 & (n-1)A = 11.6923026 & \\ (n-1)N = 12 & (n+1)N = 16 & \\ \hline 27.5897368 & : & 27.6923026 :: 1.1 \\ & & \underline{1.1} \\ & & 27.5897368 \overline{)30.46153286} \\ & & \underline{1.10408, \&c. \text{ Ans.}} \end{array}$$

(5)

Here $N=2$, $n=9$. Assume the root $= 1.1$.
 Then $A = (1.1)^9 = 2.357947691$. Also
 $(n+1)A = 23.57947691$ $(n-1)A = 18.863581528$
 $(n-1)N = 16$ $(n+1)N = 20$

$$\begin{array}{rcl} \hline 39.57947691 & : & 38.863581528 :: 1.1 \\ & & \underline{1.1} \\ & & 39.57947691 \overline{)42.7499396808} \\ & & \text{Quot.} = \underline{1.0801 \text{ Ans. nearly.}} \end{array}$$

(6)

Here $N=27455$; and $30^3=27000$; \therefore let $r=30.1$.
 Then $A = (30.1)^3 = 27270.901$, and $n=3$.

$$\begin{array}{rcl} (n+1)A = 109083.604 & (n-1)A = 54541.802 & \\ (n-1)N = 54910 & (n+1)N = 109820 & \\ \hline 163993.604 & : & 164361.802 :: 30.1 \\ & & \underline{30.1} \\ & & 164361802 \\ & & 493085406 \\ 163993.604 \overline{)4947290.2402} & & \\ & & \underline{30.167, \&c. \text{ Ans.}} \end{array}$$

ARITHMETIC PROGRESSION.

PROBLEM I.

(2)

$$\frac{19-3}{9-1} = \frac{16}{8} = 2 = \text{the common difference.}$$

(3)

$$\frac{58-3}{12-1} = \frac{55}{11} = 5 = \text{the common difference;}$$

$$\text{and } 3+8+13+18+23+28+33+38+43+48+53+58 \\ = 366 \text{ miles} = \text{distance travelled.}$$

PROBLEM II.

(2)

$$\frac{70-10}{3} + 1 = \frac{60}{3} + 1 = 21 = \text{No. of terms.}$$

(3)

$$(10-0) \div \frac{1}{10} = 100, \text{ and } 100+1 = 101 = \text{No. of terms.}$$

(4)

$$(7-\frac{1}{2}) \div \frac{1}{2} = \frac{13}{2} \times \frac{2}{1} = 13, \text{ and } 13+1 = 14 = \text{No. required.}$$

PROBLEM III.

(2)

$$(1+21) \times \frac{1}{2} = \frac{22 \times 11}{2} = 11 \times 11 = 121 = \text{sum.}$$

(3)

$$(1+24) \times \frac{1}{2} = \frac{25 \times 24}{2} = 25 \times 12 = 300 \text{ strokes.}$$

(4)

$$\text{Here } 1+1 = 2 = \text{first term.}$$

$$100+100 = 200 = \text{last term.}$$

$$\therefore (2+200) \times \frac{100}{2} = 101 \times 100 = 10100 \text{ yards} \\ = 5 \text{ miles } 1300 \text{ yards.}$$

(5)

$$(1+1000) \times \frac{100}{2} = 1001 \times 50 = 50050.$$

(6)

$$8 - \frac{1}{3} \times 26 = 8 - 8\frac{2}{3} = -\frac{2}{3} = \text{the least term;}$$

$$(8 - \frac{1}{3}) \times \frac{27}{2} = \frac{23}{3} \times \frac{27}{2} = 11 \times 9 = 99 = \text{sum.}$$

PROBLEM IV.

(2)

Common diff. = $\frac{11-1}{4} = \frac{5}{2} = 2\frac{1}{2}$; therefore the means are
 $3\frac{1}{2}, 6, 8\frac{1}{2}.$

(3)

Common diff. = $\frac{2-1}{4} = \frac{1}{4}$; therefore the means are $1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}.$

(4)

Common diff. = $\frac{3-2}{10} = \frac{1}{10}$; \therefore the means are $2\frac{1}{10}, 2\frac{2}{10}, 2\frac{3}{10},$
 $2\frac{4}{10}, 2\frac{5}{10}, 2\frac{6}{10}, 2\frac{7}{10}, 2\frac{8}{10}, 2\frac{9}{10}.$

GEOMETRICAL PROGRESSION.

PROBLEM I.

(2)

First term = $\frac{2}{3}$; common ratio = 3; \therefore the 10th term = $\frac{2}{3} \times 3^9$
 $= 29524\frac{1}{2}.$

(3)

First term = 100; common ratio = $\frac{1}{2}$; \therefore the 10th term =
 $100 \times (\frac{1}{2})^9 = \frac{100}{128}.$

PROBLEM II.

(2)

$$\frac{4 \times 65536 - 1}{4 - 1} = \frac{262144 - 1}{3} = \frac{262143}{3} = 87381 \text{ Ans.}$$

(3)

$$\text{The sum} = \frac{1\frac{1}{2} \times 59049 - 1024}{1\frac{1}{2} - 1} = \frac{87549\frac{1}{2}}{\frac{1}{2}} = 175099 \text{ Ans.}$$

(4)

The last term = $2^{25} = 4194304$; and the sum = $\frac{2 \cdot 2^{25} - 1}{2 - 1} =$
 $8388607.$

(5)

The sum of all the farthings = $\frac{2^{24}-1}{2-1} = 4294967295 =$
 $1073741823\frac{1}{4}d. = 89478485s. 3\frac{1}{4}d. = 4473924l. 5s. 3\frac{1}{4}d.$

(6)

The first term in this progression is 1, the number of terms 64 (the number of squares on the chess-board), and the common ratio is 2. Hence the sum of all the grains =
 $\frac{2^{64}-1}{2-1} = \frac{2^{64}-1}{1} = 2^{64}-1 = 18446744073709551615.$

7680 grs. in { 10) 1844674407370955161,5

1 pint. { 768) 1844674407370955161

64 pints in { 8) 2401919801264263 pints.

1 bushel. { 8) 300239975158032

4,0) 3752999689475,4 bushels.

1,0000) 938249922,368 tons.

938249922 the No. of ships req.

PROBLEM III.

(2)

$1\frac{2}{3} = 256$; $\therefore \sqrt[4]{256} = 4$, the common ratio; \therefore the three means required are 24, 96, 384.

(3)

$1\frac{1}{3} = 32$; $\therefore \sqrt[4]{32} = 2$, the common ratio; \therefore the four means required are 6, 12, 24, 48.

(4)

$1\frac{1}{3} = 81$; $\therefore \sqrt[4]{81} = 3$, the common ratio; \therefore the three means required are 9, 27, 81.

BARTER.

(2)
20 cwt. of tobacco, at 3*l.* per cwt., are worth 60*l.*, or 14400 pence; therefore

$$\frac{14400}{8} = 1800 \text{ pounds} = 16 \text{ cwt. } 8 \text{ lbs. } Ans.$$

(3)
4 cwt. 2 qrs. of chocolate, at 4*s.* per lb., are worth 2016 shillings; therefore

$$\frac{2016}{9} = 224 \text{ lb.} = 2 \text{ cwt. of tea. } Ans.$$

(4)
37 pieces, at 1*l.* 12*s.* 4*d.* each, are worth 37×1552 farthings; and 2*s.* 9½*d.* = 134 farthings; hence

$$\frac{37 \times 1552}{134} = \frac{57424}{134} = 428\frac{3}{4} \text{ reams. } Ans.$$

(5)
1000 yds., at 9½*d.* per yd., are worth 38000 farthings; and 10½*d.* = 41 farthings; hence

$$\frac{38000}{41} = 926\frac{3}{4} \text{ yards. } Ans.$$

PROFIT AND LOSS.

(2)
As 120*l.* : 100*l.* :: 1000*l.* : $\frac{1000000}{120} \text{ l.} = 833\text{i}l. \text{ } 6\text{s. } 8\text{d.}$

(3)
As 12*d.* : 100*l.* :: 1½*d.* : $\frac{100 \times 1\frac{1}{2}}{12} \text{ l.} = \frac{150}{12} \text{ l.} = 12\text{i}l. \text{ } 10\text{s.}$

(4)
As 1*l.* : 100*l.* :: 3*s.* 6*d.* : $\frac{100 \times 42}{1} = 4200\text{i}d. = 350\text{s.} = 17\text{i}l. \text{ } 10\text{s.}$

(5)
As 16*d.* : 4*d.* :: 100*l.* : $\frac{4 \times 100}{16} = \frac{100}{4} = 25\text{i}l. \text{ the gain per cent.}$

(6)
As 1 lb. : 112 lb. :: 4½*d.* : 2*l.* 2*s.* the cost price of 1 cwt.; and 2*l.* 7*s.* 4*d.* is the selling price per cwt.; hence 5*s.* 4*d.* is the gain by 1 cwt.; then

$$\text{as } 2\text{i}l. \text{ } 2\text{s.} : 5\text{s. } 4\text{d.} :: 100\text{i}l. : 12\text{i}l. \text{ } 13\text{s. } 11\frac{1}{2}\text{d. } \frac{1}{4}\text{d. } Ans.$$

(7)

The seller loses $17\frac{1}{2}$ per cent. ; $\therefore 100 - 17\frac{1}{2}$, or $82\frac{1}{2}l.$ = the sum received for each 100*l.* laid out ; hence

$$100l. : 82\frac{1}{2}l. :: 7s. 6d. : 6s. 2\frac{1}{2}d. \text{ Ans.}$$

(8)

The value of 1 cwt. at $5\frac{1}{2}d.$ per lb. = 49*s.* ;

$\therefore 49 - 42 = 7s.$ = the gain upon 2*l.* 2*s.*

$$\therefore 42s. : 7s. :: 100l. : 16l. 13s. 4d. \text{ Ans.}$$

(9)

$17\frac{1}{2}$ cwt. = 1960 lb., which cost 36*l.* 15*s.* Also the price obtained for 1960 lb. = $1960 \times 7\frac{1}{2}$ pence = 61*l.* 5*s.* ; hence the gain = 24*l.* 10*s.*, and

$$36l. 15s. : 24l. 10s. :: 100l. : 66l. 13s. 4d. \text{ Ans.}$$

(10)

As 100*l.* : 112*l.* :: 10 guineas : $\frac{112}{100}$ guinea = retail price per cwt. ; therefore

$$\frac{112}{10 \times 112}, \text{ or } \frac{1}{10} \text{ guinea, or } 2s. 1d. \frac{1}{4}f. = \text{retail price per lb.}$$

(11)

As 110 : 100 :: 7*s.* : $\frac{70}{110}s. = 6\frac{1}{11}s.$ = cost price per yard ; $\therefore 8\frac{1}{2}s. - 6\frac{1}{11}s.$, or $2\frac{11}{22}s. = \frac{11}{11}s.$ = gain per yard, when the cloth is sold at 8*s.* 6*d.*

$$\text{Also as } \frac{70}{110} : \frac{11}{11} :: 100l. : \frac{47 \times 100}{22} \times \frac{11}{70} = \frac{47 \times 10}{2 \times 7}l. =$$

$$\frac{235}{7}l. = 33l. 11s. 5\frac{1}{2}d. = \text{gain per cent.}$$

(12)

$28 \times 4 = 112l.$ = cost price of the 28 pieces.

$\frac{60l.}{10} =$ sale price of 10 pieces.

$\frac{40l.}{8} =$ sale price of 8 pieces by the question.

$\frac{12l.}{10} =$ cost price of the remaining 10 pieces.

Hence as 100*l.* : 20*l.* :: 112*l.* : 22*l.* 8*s.* the gain upon the whole ; hence 22*l.* 8*s.* + 12*l.* = 34*l.* 8*s.* = the price which the 10 pieces must be sold for, and therefore 1 of them must be sold for 3*l.* 8*s.* 9 $\frac{1}{2}d.$ $\frac{1}{4}$

(13)

Cost of 40 gallons, at 3*s.* per gallon = 6*l.* ; hence 100 : 110 :: 6*l.* : 6*l.* 12*s.* = sale price of the whole ; but 6 gallons were lost, and hence 34 remain ;

$$\therefore 34 : 1 :: 6l. 12s. : \text{sale price per gall.} = 3s. 10\frac{1}{2}d. \frac{1}{4}f.$$

(14)

Since 12 per cent. are lost by remittance, therefore

$100 : 88 :: 6s. : 5s. 3\frac{1}{2}d.$ = sterling value per pair ;

but $4s. 3d. + 2d. = 4s. 5d.$ = real cost per pair ;

$10\frac{1}{2}d.$ = gain on $4s. 5d.$

$\therefore 4s. 5d. : 10\frac{1}{2}d. :: 100l. : \text{gain} = \frac{1036}{53}l. = 19l. 10s. 11\frac{1}{2}d.$

(15)

As $100 - 17 : 100 + 20 :: 50$ guineas, or

$83 : 120 :: 52l. 10s. : \frac{6300}{83}l. = 75l. 18s. 0\frac{1}{2}d. \frac{1}{3}$, the trade

price ; hence

$75l. 18s. 0\frac{1}{2}d. - 52l. 10s. = 23l. 8s. 0\frac{1}{2}d.$, below the value.

PARTNERSHIP.

CASE I.

(2)

Total stock = A's money + B's money = $300l. + 400l. = 700l.$; hence by the rule

$700 : 300 :: 182$ (the gain) : A's share = $78l.$ } *Ans.*
Also $700 : 400 :: 182$: B's share = $104l.$

(3)

The sum of $120l.$ must evidently be divided into 6 equal parts, one of which is the share of one person, two parts make the share of another, and three parts form the share of the third person. Hence

$6 : 1 :: 120 : 20$ = first person's share.

$6 : 2 :: 120 : 40$ = second person's share.

$6 : 3 :: 120 : 60$ = third person's share.

(4)

A's stock = $184l. 10s. = 3690s.$

$220l. 12s.$

B's ,, = $96 15 = 1935$

20

C's ,, = $76 5 = 1525$

4412s. = gain.

\therefore whole stock = $357 10 = 7150$; hence

$7150 : 4412 :: 3690s. : 113l. 16s. \frac{4}{5}$ = A's share of profit.

$7150 : 4412 :: 1935 : 59l. 14s. \frac{1}{5}$ = B's share.

$7150 : 4412 :: 1525 : 47l. 1s. \frac{1}{5}$ = C's share.

(5)

Total stock in trade = 1200*l.* Total gain = 3600*l.*

1200 : 3600 :: 180	(A's stock)	: 540 = A's share of profit.
1200 : 3600 :: 240	(B's stock)	: 720 = B's share of do.
1200 : 3600 :: 350	(C's stock)	: 1050 = C's share of do.
1200 : 3600 :: 430	(D's stock)	: 1290 = D's share of do.

(6)

A loaded 110 tons	} ∴ C loaded 133 tons.
B loaded 97 tons	

Hence 340 : 85 (total loss) :: 118 : 27½ = A's loss.
 340 : 85 :: 97 : 24½ = B's loss.
 340 : 85 :: 133 : 33½ = C's loss.

(7)

¼th = A's share of the ship	} ∴ ¼ = C's share.
¼th = B's share of the ship	

Now 500*l.* were insured, hence total loss to owners = 360*l.*Hence the whole ship : ¼th of her :: 360*l.* : A's loss.

Or 1 : ¼ :: 360 : 45 = A's loss.

And 1 : ½ :: 360 : 90 = B's loss.

1 : ⅔ :: 360 : 225 = C's loss.

(8)

Debt due to A	= 275	14 = 5514
„ B	= 304	7 = 6087
„ C	= 152	0 = 3040
„ D	= 104	6 = 2086

Total debts of the bankrupt = $\frac{836}{7} = 16727$ The assets are 675*l.* 15*s.* = 13515*s.*; hence

16727 : 13515 :: 5514	: 222	15	2 = A's dividend.
16727 : 13515 :: 6087	: 245	18	1½ = B's dividend.
16727 : 13515 :: 3040	: 122	16	2½ = C's dividend.
16727 : 13515 :: 2086	: 84	5	5 = D's dividend.

(9)

By the nature of this question, A had 3 per cent. allowed for his trouble; hence, since A is to have 8 parts where B receives only 5, the whole gain is to be considered as divided into 13 parts, whereof A gets three for his trouble:

$$\therefore 13 : 3 :: 154*l.* : \frac{154 \times 3}{13} *l.* = \frac{462}{13} *l.* = 35*l.* 10*s.* 9½*d.*$$

(10)

The whole stock = 10050*l.*; whole gain = 10,000*l.*;

$$\begin{array}{l}
 \therefore 10050 : 10000 :: 2000 : 1990 \quad 0 \quad 11\frac{1}{2} = \text{A's share.} \\
 10050 : 10000 :: 3500 : 3482 \quad 11 \quad 8\frac{1}{2} = \text{B's share.} \\
 10050 : 10000 :: 4550 : 4527 \quad 7 \quad 3 = \text{C's share.}
 \end{array}$$

CASE II.

(2)

$$7 \times 3 = 21 = \text{A's stock} \times \text{time.}$$

$$9 \times 5 = 45 = \text{B's stock} \times \text{time.}$$

$$4 \times 12 = 48 = \text{C's stock} \times \text{time.}$$

$$\text{Sum} = \underline{114}$$

$$\begin{array}{l}
 \text{Hence } 114 : 30 :: 21 : 5 \quad 10 \quad 6\frac{1}{2} \quad \frac{30}{114} = \text{A's portion.} \\
 114 : 30 :: 45 : 11 \quad 16 \quad 10 \quad \frac{45}{114} = \text{B's portion.} \\
 114 : 30 :: 48 : 12 \quad 12 \quad 7\frac{1}{2} \quad \frac{48}{114} = \text{C's portion.}
 \end{array}$$

(3)

$$5 \times 4\frac{1}{2} = 22\frac{1}{2} = \text{A's stock} \times \text{time.}$$

$$8 \times 5 = 40 = \text{B's ditto.}$$

$$9 \times 6\frac{1}{2} = 58\frac{1}{2} = \text{C's ditto.}$$

$$\text{Sum} = \underline{121}$$

$$\begin{array}{l}
 \therefore 121 : 1210 :: 22\frac{1}{2} : 11 \quad 5 = \text{A's portion.} \\
 121 : 1210 :: 40 : 20 \quad 0 = \text{B's portion.} \\
 121 : 1210 :: 58\frac{1}{2} : 29 \quad 5 = \text{C's portion.}
 \end{array}$$

(4)

(A)

$$200 \times 18 = 3600$$

$$100 \times 10 = 1000$$

$$\underline{4600}$$

$$7940$$

(B)

$$550 \times 18 = 9900$$

$$140 \times 14 = 1960$$

$$\underline{7940}$$

$$\begin{array}{l}
 12540 : 4600 :: 526 : 192 \quad 19 \quad 0 = \text{A's gain.} \\
 12540 : 7940 :: 526 : 333 \quad 0 \quad 11\frac{1}{2} = \text{B's gain.}
 \end{array}$$

(5)

They traded a whole year by the question ;

$$\therefore 1000 \times 12 = \text{A's stock} \times \text{time} = 12000$$

$$1500 \times 10 = \text{B's stock} \times \text{time} = 15000$$

$$2800 \times 7 = \text{C's stock} \times \text{time} = 19600$$

$$\text{Sum} = \underline{46600}$$

$$46600 : 1776 \text{ } 10 :: 12000 : 457 \text{ } 9 \text{ } 4\frac{1}{2} = \text{A's share.}$$

$$46600 : 1776 \text{ } 10 :: 15000 : 571 \text{ } 16 \text{ } 8\frac{1}{2} = \text{B's share.}$$

$$46600 : 1776 \text{ } 10 :: 19600 : 747 \text{ } 3 \text{ } 11\frac{1}{2} = \text{C's share.}$$

ALLIGATION.

CASE I.

(2)

lb.	s.	d.	s.	d.
5	×	(7	0)	= 35 0
9	×	(8	6)	= 76 6
14½	×	(5	10)	= 84 7
28½		28½)	196 1
		<i>Ans. 6s. 19½d.</i>		

(3)

gall.	s.	d.	s.	d.
4	×	(4	10)	= 19 4
7	×	(5	3)	= 36 9
9½	×	(5	8)	= 55 3
20½		20½)	111 4
		<i>Ans. 5s. 4½d.</i>		

(4)

bush.	s.	d.	s.	d.
3	×	(8	5)	= 25 3
4	×	(7	6)	= 30 0
5	×	(4	10)	= 24 2
12		12)	79 5
		<i>Ans. 6s. 7½d. ⅔</i>		

(5)

bush.	s.	d.	s.	d.
20	×	5		= 100 0
36	×	3		= 108 0
40	×	2		= 80 0
96		96)	288 0
		<i>Ans. 3s. 0d.</i>		

(6)

lb.	oz.	d.	s.
14	×	(8	5½)
18	×	(12	8½)
21		21)
		<i>Ans. 16½ carats fine.</i>	

(7)

20	×	10	= 200
18	×	16	= 288
26		26)
		<i>Ans. 18½ carats fine.</i>	

Now standard gold is 22 carats fine ; $\therefore 22 - 18\frac{1}{2} = 3\frac{1}{2}$
the *worseness** of the mixture. Hence no alloy must be
put in, but more gold.

* In the language of an assay master.

CASE II.

(2)

15s. { proposed price } 2 { Hence a mixture of 2 gall. at
 18s. { 16s. } 1 { 15s., and 1 gall. at 18s. per gall.,
 will be worth 16s. per gall.

(3)

46 { 30 } 10 gallons at 2s. 6d.
 { 44 } 2 gallons at 3s. 8d.
 { 48 } 2 gallons at 4s. 0d.
 { 56 } 16 gallons at 4s. 8d.

Or thus: 46 { 30 } 10 + 2 = 12 gallons at 2s. 6d.
 { 44 } 10 + 2 = 12 gallons at 3s. 8d.
 { 48 } 16 + 2 = 18 gallons at 4s. 0d.
 { 56 } 16 + 2 = 18 gallons at 4s. 8d.

(4)

21 { 17 } 3 at 17 carats.
 { 18 } 1 at 18 carats:
 { 22 } 3 at 22 carats.
 { 24 } 4 at 24 carats.

(5)

5 { 8 } 5 at 8s. per gallon.
 { 7 } 4 at 7s. per gallon.
 { 1 } 2 at 1s. per gallon.
 { 0 } 3 of water.

Or, 9 gallons at 8s., 9 gallons at 7s., 5 gallons at 1s.,
 and 5 of water.*

(6)

7 { 4 } 4 lbs. at 4d. per lb.
 { 6 } 4 lbs. at 6d. per lb.

3 + 1 = 4 lbs. at 11d. Or 1 lb.
 of each sort will make the required mixture; or any other
 equal portion of each.

(7)

9 { 16 } 3 gallons at 16s. per gallon.
 { 12 } 4 gallons at 12s. per gallon.
 { 6 } 7 gallons at 6s. per gallon.
 { 5 } 3 gallons at 5s. per gallon.

* These questions belong to that branch of analysis called *indeterminate*, because they admit a variety of answers.

(8)

$$\begin{array}{rcl}
 160 \left\{ \begin{array}{l} 108 \\ 126 \\ 144 \\ 228 \end{array} \right. & \begin{array}{l} 68 \text{ at } 9s. \text{ per lb.} \\ 68 \text{ at } 10s. 6d. \text{ per lb.} \\ 68 \text{ at } 12s. \text{ per lb.} \\ 52 + 34 + 16 = 102 \text{ lb. at } 19s. \text{ per lb.} \end{array}
 \end{array}$$

Or it is evident that the halves of these several quantities will compose a mixture worth 19s. per lb. Hence 34 lb. at 9s., 34 lb. at 10s. 6d., 34 lb. at 12s., and 51 lb. at 19s., will compose a mixture of the rate required.

CASE III.

(2)

$$\begin{array}{rcl}
 8 \left\{ \begin{array}{l} 4 \\ 6 \\ 9 \\ 11 \end{array} \right. & \begin{array}{l} 3 \\ 1 \\ 2 \\ 4 \end{array} & \begin{array}{l} 10 : 240 :: 3 : 72 \text{ lbs. at } 4d. \\ 10 : 240 :: 1 : 24 \text{ lbs. at } 6d. \\ 10 : 240 :: 2 : 48 \text{ lbs. at } 9d. \\ 10 : 240 :: 4 : 96 \text{ lbs. at } 11d. \end{array} \\
 & \underline{10} &
 \end{array}$$

(3)

$$\begin{array}{rcl}
 17 \left\{ \begin{array}{l} 12 \\ 15 \\ 18 \end{array} \right. & \begin{array}{l} 1 = 1 \\ 1 = 1 \\ 5 + 2 = 7 \end{array} & \begin{array}{l} 9 : 1000 :: 1 : 111\frac{1}{3} \text{ gall. at } 12s. \\ 9 : 1000 :: 1 : 111\frac{1}{3} \text{ gall. at } 15s. \\ 9 : 1000 :: 7 : 777\frac{1}{3} \text{ gall. at } 18s. \end{array} \\
 & \underline{9} &
 \end{array}$$

CASE IV.

(2)

$$\begin{array}{rcl}
 15s. 4d. = 184d. & \begin{array}{l} d. \\ \left\{ \begin{array}{l} 160 \\ 150 \\ 168 \\ 198 \end{array} \right. \end{array} & \begin{array}{l} 14 \text{ at } 13s. 4d. \text{ per gallon.} \\ 14 \text{ at } 12s. 6d. \text{ per gallon.} \\ 14 \text{ at } 14s. \text{ per gallon.} \\ 24 + 34 + 16 = 74 \text{ at } 16s. 6d. \text{ per } \text{[gall.]} \end{array}
 \end{array}$$

Then as 14 : 14 :: 500 : 500 at 13s. 4d. per gallon.
 14 : 14 :: 500 : 500 at 12s. 6d. per gallon.
 14 : 14 :: 500 : 500 at 14s. per gallon.
 14 : 74 :: 500 : 2642\frac{1}{2} at 16s. 6d. per gallon.

$$\begin{array}{rcl}
 & & (3) \\
 20 \left\{ \begin{array}{l} 15 \\ 17 \\ 22 \\ 18 \end{array} \right. & \begin{array}{l} 2 \\ 2 \\ 5+3+2 \\ 2 \end{array} & \begin{array}{l} = 2 \text{ at 15 carats.} \\ = 2 \text{ at 17 carats.} \\ = 10 \text{ at 22 carats.} \\ = 2 \text{ at 18 carats.} \end{array}
 \end{array}$$

Hence $2 : 5 :: 2 : 5$ oz. of 15 carats fine. }
 $2 : 5 :: 2 : 5$ oz. of 17 carats fine. } *Ans.*
 $2 : 5 :: 10 : 25$ oz. of 22 carats fine. }

SINGLE POSITION.

(2)
 Suppose the sum = 480*l.*; then

$$\begin{array}{rcl}
 \frac{1}{3} \text{ of } 480 & = & 160 \\
 \frac{1}{4} \text{ of } 480 & = & 120 \\
 \frac{1}{6} \text{ of } 480 & = & 80 \\
 \hline
 \text{Sum} & = & 360 \\
 \text{Rem.} & = & 120
 \end{array}$$

As $120 : 80 :: 480*l.* : 320*l.*$ *Ans.*

(3)
 Suppose he had 120*l.*; then

$$120 - \frac{120}{3} - \frac{120}{4} = 120 - 40 - 30 = 50*l.* \text{ left; } ^{\circ} \\
 \therefore 5 : 60 :: 12*l.* : 144*l.* \text{ } \textit{Ans.}$$

(4)
 Suppose the number = 24; then

$$24 + \frac{24}{2} + \frac{24}{3} + \frac{24}{4} = 24 + 12 + 8 + 6 = 50; \\
 \therefore 50 : 125 :: 24 : 60 = \text{No. required.}$$

(5)
 Suppose the horse cost 6*l.*
 Then the harness cost 3
 And the chaise cost 18

Total cost = 27

$$\begin{array}{rcl}
 \therefore 27 : 60 :: \overset{\text{£}}{6} : \overset{\text{£}}{13} & \overset{\text{£}}{6} & \overset{\text{£}}{8} = \text{cost of horse.} \\
 27 : 60 :: \overset{\text{£}}{3} : \overset{\text{£}}{6} & \overset{\text{£}}{13} & \overset{\text{£}}{4} = \text{cost of harness.} \\
 27 : 60 :: \overset{\text{£}}{18} : \overset{\text{£}}{40} & \overset{\text{£}}{0} & \overset{\text{£}}{0} = \text{cost of chaise.} \\
 \text{Proof} & \underline{\underline{60}} & \underline{\underline{0}} \quad \underline{\underline{0}}
 \end{array}$$

(6)

Here the 1st cock can fill 1 vessel in 1 hour.
 the 2nd „ $\frac{1}{2}$ vessel in 1 hour.
 the 3rd „ $\frac{1}{3}$ vessel in 1 hour.

When all the cocks are open at the same time, they can fill $1\frac{1}{6}$ vessels in 1 hour.

Hence $1\frac{1}{6} : 1 :: 60 \text{ min.} : 60 \div 1\frac{1}{6} = 32\frac{2}{3} \text{ min.} = 32\frac{2}{3}^{\circ}.$ *

(7)

Suppose he had 400 sheep; then

$$400 + 400 + 200 + 7\frac{1}{2} = 1007\frac{1}{2};$$

then $1007\frac{1}{2} : 400 :: 1000 : 397 \text{ sheep.}^{\dagger} \text{ Ans.}$

(8)

Suppose 180 to be the number required; then

$$\frac{180}{3} + \frac{180}{4} + \frac{180}{5} = 60 + 45 + 36 = 141;$$

and $141 : 180 :: 94 : 120 \text{ Ans.}$

DOUBLE POSITION, p. 133.

(3)

Let the two suppositions be 140 and 147; then

$$\left(\frac{140}{7} - 10\right) \times 3 = 10 \times 3 = 30 = \text{first result};$$

$$\left(\frac{147}{7} - 10\right) \times 3 = 11 \times 3 = 33 = \text{second result.}$$

Now $33 - 30 = 3 = \text{difference of results};$

$147 - 140 = 7 = \text{difference of suppositions};$

$30 - 24 = 6 = \text{difference between true and first result};$

$\therefore 3 : 7 :: 6 : 14 = \text{correction};$ and

$140 - 14 = 126 = \text{the number required.}$

* This question does not belong to the Rule of Position.

\dagger 397 is the nearest integral value obtained from the proportion; but there is no supposition which we ought to use that will give the answer without remainder.

(4)

Let 2.5 and 2.6 be the suppositions; then

$$\begin{array}{r} 2.5^3 = 15.625 \quad 2.6 \quad 17 \\ 2.6^3 = 17.576 \quad 2.5 \quad 15.625 \end{array}$$

$$1.951 : .1 :: 1.375 : .07;$$

whence $2.5 + .07 = 2.57$ = the cube root to two places of decimals. If 2.57 and 2.58 be the two new suppositions, and the process repeated, the cube root will be found = 2.571282.

(5)

Let the two suppositions be 68 and 76; then

$$(68 \times 3 + 4) \div 8 = \frac{208}{8} = 26 = \text{first result};$$

$$(76 \times 3 + 4) \div 8 = \frac{232}{8} = 29 = \text{second result}.$$

$$\text{Then } 29 - 26 : 76 - 68 :: 32 - 29 : \text{correction} = \frac{3 \times 8}{3} = 8;$$

$$\text{hence } 76 + 8 = 84 = \text{number required}.$$

(6)

Let their income be 80% per annum; then

(A)	(B)
80	80
$\frac{1}{3}$ of 80 = 16 saved.	$64 + 50 = 114$ spends.
64 spends.	34 debt in 1 year.
	4

$$\text{1st result} = \frac{136}{4} \text{ debt in 4 years.}$$

Let their income be 90% per annum; then

90	
$\frac{1}{3}$ of 90 = 18	90
72 + 50 = 122	what B spends.
32 debt in 1 year.	
4	

$$\text{2nd result} = \frac{128}{4} \text{ debt in 4 years.}$$

As $136 - 128 : 90 - 80 :: 128 - 100 : \text{correction};$ or, $8 : 10 :: 28 : 35 = \text{correction}.$ $\therefore 90 + 35 = 125\% = \text{their income}.$

$$\frac{1}{3} \text{ of } 125 = 25 \text{ saved by A.}$$

$$\frac{100}{100} \text{ spent by A per annum.}$$

$$\frac{50}{50}$$

$$\frac{150}{150} \text{ spent by B per annum.}$$

(7)

Suppose the number to be 218l.; then

$$218 + 126 = \dots\dots\dots 344$$

$$(218 - 87) \times 2 = 131 \times 2 = 262$$

$$\text{Correction} = 82$$

$$\text{Supposed No.} = 218$$

$$\text{True No.} = 300\text{l.}$$

A second supposition is not required, as any one supposition will answer the question. Indeed no supposition at all is required; for

$$126 + 87 \times 2 = 126 + 174 = 300 \text{ Ans.}$$

(8)

Suppose he worked 28 days, and was idle 12; 2l. 1s. 8d. = 500d., and therefore

$$28 \times 20 = 560 = \text{sum earned.}$$

$$12 \times 10 = 120 = \text{sum forfeited.}$$

$$440 = \text{sum to receive.}$$

$$\text{But by the question } 500 = \text{sum received.}$$

$$60 = 1\text{st result.}$$

Let us suppose he worked 31 days, and was idle 9; then

$$31 \times 20 = 620 = \text{sum earned.}$$

$$9 \times 10 = 90 = \text{sum forfeited.}$$

$$530$$

$$500$$

$$30 = 2\text{nd result.}$$

Now it is evident that the 1st supposition is too small and the second too great; hence we must take the *sum* instead of the *difference* of the results; and

$$60 + 30 : 31 - 28 :: 30 : \text{correction};$$

$$\therefore \text{correction} = \frac{30 \times 3}{90} = 1; \text{ and } 31 - 1 = 30 \text{ days};$$

hence he was idle 10 days, and worked 30 days.

Otherwise, thus: For every day he was idle he lost 30d.; whereas he ought to have earned 20d.; hence for every idle day he, in effect, lost 50d.; and he lost in all 2l. 1s. 8d., or 500d.; therefore he was idle $500 \div 50 = 10$ days.

Or we may reason in this manner: Had he worked

every day he would have earned $40 \times 20 = 800d.$; but he received only $500d.$, and lost $300d.$ Now he lost $30d.$ for every idle day, and $300 \div 30 = 10 =$ number of days he was idle.

(9)

Let the value of the worse horse be $28l.$; then $(28 + 50) \div 2 = 78 \div 2 = 39l. =$ value of the other horse;

Hence $39 + 50 = 89$, and $28 \times 3 = 84$; therefore
 $89 - 84 = 5 = 1st \text{ result.}$

Again, let $32l.$ be the value of the first horse; then $(32 + 50) \div 2 = 82 \div 2 = 41l. =$ value of the second horse; consequently

$(41 + 50) - 3 \times 32 = 91 - 96 = -5 = 2nd \text{ result};$

$\therefore 5 + 5 : 32 - 28 :: 5 : \text{correction};$

$\therefore \text{correction} = \frac{5 \times 4}{10} = 2$, and $28 + 2 = 30l. =$ value of the one horse, and $\frac{1}{2} (30 + 50) = 40l. =$ the value of the other.

(10)

Let 40 inches be the length of his body; then his tail = $9 + \frac{1}{2}$ of $40 = 29$ inches; and therefore

$40 - (9 + 29) = 40 - 38 = 2 = 1st \text{ result.}$

Let 44 inches be the length of his body; then his tail = $9 + \frac{1}{2}$ of $44 = 31$ inches; and

$44 - (9 + 31) = 44 - 40 = 4 = 2nd \text{ result};$

$\therefore 4 - 2 : 44 - 40 :: 2 : \text{correction} = \frac{2 \times 4}{2} = 4;$

Hence $40 - 4 = 36$ inches = length of body.

$\therefore 9 + \frac{1}{2}$ of $36 = 27$ inches = length of head.

9 inches = length of tail.

72 inches = 6 feet = whole length of fish.

PERMUTATIONS AND COMBINATIONS.*

PROBLEM I.

(2)

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$, the number of different positions which 7 persons can occupy at table = number of days required.

(3)

Here $1.2.3.4.5.6.7.8.9.10.11.12 = 479001600$ changes, and this divided by 10 gives 47900160 minutes. Now 365 days 5 hrs. 49 min. = 525949 min., and

$$\frac{47900160}{525949} = 91 \text{ yrs. } 38801 \text{ min.}$$

$$= 91 \text{ yrs. } 26 \text{ days, } 22 \text{ hrs. } 41 \text{ min.}$$

(4)

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 = 40320 \text{ changes.}$$

PROBLEM II.

(2)

$$\frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}{1 \times 2 \times 3 \times 4 \times 1 \times 2 \times 3 \times 1 \times 2} = 12600 \text{ Ans.}$$

(3)

$$\frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8}{1 \times 2 \times 3 \times 1 \times 2} = 3360 \text{ Ans.}$$

PROBLEM III.

(2)

$$26 \times 25 \times 24 \times 23 = 358800 \text{ Ans.}$$

(3)

$$10 \times 9 \times 8 \times 7 \times 6 \times 5 = 151200 \text{ Ans.}$$

PROBLEM IV.

(2)

$$\frac{24 \times 23}{1 \times 2} = 12 \times 23 = 276 \text{ Ans.}$$

* For a theoretical view of this subject, see Tyson's Edition of "Bonycastle's Algebra."

$$\begin{array}{r}
 (3) \\
 100 \cdot 99 \cdot 98 \cdot \dots \cdot 91 \\
 \hline
 1 \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot 10 \text{ farthings.} \\
 = 4327577364110 \text{ pence.} \\
 12 \overline{) 2,0)36063144700,9} \text{ 2d.} \\
 \hline
 \text{£}18031572350 \text{ 9 2 the reward req.}
 \end{array}$$

PROBLEM V.

$$\begin{array}{r}
 (2) \\
 2^8 - 1 = 64 - 1 = 63 \text{ Ans.}
 \end{array}$$

PROBLEM VI.

$$\begin{array}{r}
 (2) \\
 13 \times \frac{13^{10} - 1}{13 - 1} = 328114698808273 \text{ Ans.}
 \end{array}$$

COMMISSION, BROKERAGE, &c.

$$\begin{array}{r}
 (2) \\
 610 \times \frac{1}{4} \div 100 = \frac{152 \text{ l. } 10 \text{ s.}}{100} = 1 \text{ l. } 10 \text{ s. } 6 \text{ d.}
 \end{array}$$

$$\begin{array}{r}
 (3) \\
 \begin{array}{r}
 \text{£.} \quad \text{s.} \quad \text{d.} \\
 4 \overline{) 529 \text{ 18} \quad 5 \times 2} \\
 \underline{132 \quad 9 \quad 7\frac{1}{2}} \\
 1059 \text{ 16 10} \\
 \text{£}11,92 \text{ 6 } 5\frac{1}{2} \\
 \underline{20} \\
 \text{s. } 18,46 \\
 \underline{12} \\
 \text{d. } 5,57 \\
 \underline{4} \\
 \text{f. } 2,29 \\
 \text{Ans. } \text{£}11 \text{ 18s. } 5\frac{1}{2}\text{d.}
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 (4) \\
 \begin{array}{r}
 \text{£.} \\
 1200 \\
 \underline{7} \\
 8400 \\
 \frac{1}{4} = 600 \\
 \frac{1}{4} \text{ of } \frac{1}{4} = 150 \\
 \text{£}91,50 \\
 \underline{20} \\
 \text{s. } 10,00 \\
 \text{Ans. } \text{£}91 \text{ 10s.}
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 (5) \\
 \begin{array}{r}
 \text{£.} \\
 950 \\
 \underline{4} \\
 3800 \\
 \frac{1}{4} = 118 \text{ 15} \\
 \text{£}36,81 \text{ 5} \\
 \underline{20} \\
 \text{s. } 16,25 \\
 \underline{12} \\
 \text{d. } 3,00 \\
 \text{Ans. } \text{£}36 \text{ 16s. } 3\text{d.}
 \end{array}
 \end{array}$$

<div>(6) £ s. d. 876 5 10 3 ----- 2628 17 6 ½ = 438 2 11 ⅓ of ½ = 219 1 5½ £32,86 1 10½ 20 ----- s. 17,21 12 ----- d. 2,62 4 ----- f. 2,50</div>	<div>(7) £ s. 879 18 ½ = 219 19 6 ⅓ of ½ = 109 19 9 £3,29 19 3 20 ----- s. 5,99 12 ----- d. 11,91 4 ----- f. 3,64</div>	<div>(8) £ s. d. 508 17 10 ½ = 254 8 11 £7,63 6 9 20 ----- s. 12,66 12 ----- d. 8,01</div>
<div>(9) £ s. d. 1087 15 6½ ½ = 543 17 9½ ⅓ of ½ = 185 19 5½ £17,67 12 9 20 ----- s. 13,52 12 ----- d. 6,33 4 ----- f. 1,32</div>	<div>(10) £ s. 1050 ½ = 525 ⅓ of ½ = 131 5 £6,56 5 20 ----- s. 11,25 12 ----- d. 3,00</div>	<div>(11) £ s. d. 729 10 6 ½ = 182 7 7½ £9,11 18 1½ 20 ----- s. 2,38 12 ----- d. 4,59 4 ----- f. 2,38</div>
<div>(12) £ 900 10 ----- 9000 ½ = 450 ⅓ of ½ = 225 £96,75 20 ----- s. 15,00</div>	<div>(13) £ 1200 7 ----- 8400 ½ = 600 ⅓ of ½ = 150 £91,50 20 ----- s. 10,00</div>	<div>(14) £ s. d. 35727 17 6 6 × 3 = 18 ----- 214367 5 0 3 ----- 643101 15 0 ½ = 4465 19 8½ sub. £6386,35 15 3½ 20 ----- s. 7,15 12 ----- d. 1,83 4 ----- f. 3,35</div>

(15)	(16)	(17)
$\begin{array}{r} \text{£} \\ 500 \times 1 \\ \hline 12 \\ 6000 \\ 5 \\ \hline 30000 \\ 500 \\ \hline \frac{1}{4} = 250 \\ \frac{1}{4} \text{ of } \frac{1}{4} = 62 \text{ } 10 \\ \hline \text{£} 308,12 \text{ } 10 \\ 20 \\ \hline \text{s. } 2,50 \\ 12 \\ \hline \text{d. } 6,00 \end{array}$	$\begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 156 \text{ } 15 \text{ } 0 \times 2 \\ \hline 12 \\ 1881 \text{ } 0 \text{ } 0 \\ \hline 6 \\ 11286 \text{ } 0 \text{ } 0 \\ 313 \text{ } 10 \text{ } 0 \\ \hline \frac{1}{4} = 78 \text{ } 7 \text{ } 6 \\ \hline \text{£} 116,77 \text{ } 17 \text{ } 6 \\ 20 \\ \hline \text{s. } 15,57 \\ 12 \\ \hline \text{d. } 6,90 \\ 4 \\ \hline \text{f. } 3,60 \end{array}$	$\begin{array}{r} \text{£} \quad \text{s.} \\ 816 \text{ } 12 \times 1 \\ \hline 11 \\ 8982 \text{ } 12 \\ 8 \\ \hline 71860 \text{ } 16 \\ 816 \text{ } 12 \\ \hline \frac{1}{4} = 204 \text{ } 3 \\ \hline \frac{1}{4} \text{ of } \frac{1}{4} = 102 \text{ } 1 \text{ } 6 \\ \hline \text{£} 729,83 \text{ } 12 \text{ } 6 \\ 20 \\ \hline \text{s. } 16,72 \\ 12 \\ \hline \text{d. } 8,70 \\ 4 \\ \hline \text{f. } 2,80 \end{array}$

(18)	(19)	(20)
$\begin{array}{r} \text{£} \quad \text{s.} \\ 987 \text{ } 15 \times 4 \\ \hline 10 \\ 9877 \text{ } 10 \\ 11 \\ \hline 108652 \text{ } 10 \\ 3951 \text{ } 0 \\ \hline 112603 \text{ } 10 \\ \frac{1}{4} = 123 \text{ } 9 \text{ } 4\frac{1}{4} \\ \hline \text{£} 1124,80 \text{ } 0 \text{ } 7\frac{1}{4} \\ 20 \\ \hline \text{s. } 16,00 \\ 12 \\ \hline \text{d. } 0,07 \end{array}$	$\begin{array}{r} \text{£} \\ 650 \\ 10 \\ \hline 6500 \\ 9 \\ \hline 58500 \\ \frac{1}{4} = 325 \\ \hline \text{£} 588,25 \\ 20 \\ \hline \text{s. } 5,00 \end{array}$	$\begin{array}{r} \text{£} \\ 2400 \\ 12 \\ \hline 28800 \\ 7 \\ \hline 201600 \\ \frac{1}{4} = 300 \\ \hline \text{£} 2019,00 \end{array}$

SIMPLE INTEREST.

$$\begin{array}{r}
 \text{(2)} \\
 \begin{array}{r}
 \text{£} \quad \text{s.} \\
 230 \quad 10 \\
 \hline
 4 \\
 9,22 \quad 0 \\
 20 \\
 \hline
 4,40 \\
 12 \\
 \hline
 4,80 \\
 4 \\
 \hline
 3,20
 \end{array}
 \end{array}$$

Ans. £9 4s. 4½d.

$$\begin{array}{r}
 \text{(3)} \\
 \begin{array}{r}
 \text{£} \quad \text{s.} \\
 547 \quad 15 \\
 \hline
 5 \\
 2738 \quad 15 \\
 \hline
 8 \\
 82,16 \quad 5 \\
 20 \\
 \hline
 3,25 \\
 12 \\
 \hline
 3,00
 \end{array}
 \end{array}$$

Ans. £82 3s. 3d.

$$\begin{array}{r}
 \text{(4)} \\
 \begin{array}{r}
 \text{£} \quad \text{s.} \\
 690 \quad 93 \quad 3 = \text{int.} \\
 4\frac{1}{2} \quad 690 \quad 0 = \text{prin.} \\
 \hline
 2760 \quad 783 \quad 3 = \text{amt.} \\
 \hline
 345 \\
 3105 \\
 3 \\
 \hline
 93,15 \\
 20 \\
 \hline
 3,00
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{(5)} \\
 \begin{array}{r}
 \text{£} \quad \text{s.} \\
 205 \quad 15 \\
 \hline
 4 \\
 4)823 \quad 0 \\
 2,05 \quad 15 \\
 20 \\
 \hline
 1,15 \\
 12 \\
 \hline
 1,80 \\
 4 \\
 \hline
 3,20
 \end{array}
 \end{array}$$

Ans. £2 1s. 1½d.

$$\begin{array}{r}
 \text{(6)} \\
 \begin{array}{r}
 \text{£} \quad \text{s.} \\
 120 \quad 10 \\
 \hline
 4\frac{1}{2} \\
 482 \quad 0 \\
 \frac{1}{2} = 60 \quad 5 \\
 \frac{1}{2} = 30 \quad 2 \quad 6 \\
 \hline
 572 \quad 7 \quad 6 \\
 \hline
 2\frac{1}{2} \\
 1144 \quad 15 \quad 0 \\
 \frac{1}{2} = 286 \quad 3 \quad 9 \\
 \hline
 14,30 \quad 18 \quad 9
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 14,30 \quad 18 \quad 9 \\
 \hline
 20 \\
 6,18 \\
 12 \\
 \hline
 2,25 \\
 4 \\
 \hline
 1,00
 \end{array} \\
 \begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 14 \quad 6 \quad 2\frac{1}{2} = \text{interest.} \\
 120 \quad 10 \quad 0 = \text{principal.} \\
 \hline
 134 \quad 16 \quad 2\frac{1}{2} = \text{amount.}
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{(7)} \\
 \begin{array}{r}
 \text{£} \quad \text{s.} \\
 47 \quad 10 \\
 \hline
 4\frac{1}{2} \\
 190 \quad 0 \\
 \frac{1}{2} = 23 \quad 15 \\
 \hline
 2,13 \quad 15 \\
 20 \\
 \hline
 2,75 \quad 0 \\
 12 \\
 \hline
 9,00
 \end{array}
 \end{array}$$

As 365 days : 52 days :: 2l. 2s. 9d. : 6s. 1d.

8 11 0 = interest for 4 years.

∴ 8 11 0 = interest for 4 years.

0 6 1 = interest for 52 days.

8 17 1 = interest required.

(8)

$\begin{array}{r} \text{£} \\ 210 \\ 4 \\ \hline 840 \\ \frac{1}{4} = 105 \\ 9,45 \\ 20 \\ \hline 9,00 \end{array}$	<p>As 12 mo. : 7 mo. :: 9<i>l.</i> 9<i>s.</i> : 5<i>l.</i> 10<i>s.</i> 3<i>d.</i> As 365 days : 25 days :: 9<i>l.</i> 9<i>s.</i> : 0<i>l.</i> 12<i>s.</i> 11½<i>d.</i></p>
	$\begin{array}{r} \text{£} \quad \text{s} \quad \text{d} \\ 9 \quad 9 \quad 0 = \text{interest for 1 year.} \\ \hline 4 \\ 37 \quad 16 \quad 0 = \text{interest for 4 years.} \\ 5 \quad 10 \quad 3 = \text{interest for 7 months.} \\ 0 \quad 12 \quad 11\frac{1}{2} = \text{interest for 25 days.} \\ \hline 43 \quad 19 \quad 2\frac{1}{2} = \text{interest required.} \\ 210 \quad 0 \quad 0 = \text{principal.} \\ \hline 253 \quad 19 \quad 2\frac{1}{2} = \text{amount.} \end{array}$

(9)

$\begin{array}{r} \text{£} \quad \text{s} \\ 558 \quad 15 \\ 4 \\ \hline 22,35 \quad 0 \\ 20 \\ \hline 7,00 \end{array}$	<p>As $\begin{array}{r} \text{days.} \quad \text{days.} \\ 365 : 21 :: 22 \quad 7 \\ \hline 3 \\ 67 \quad 1 \\ \hline 7 \\ 365)469 \quad 7(1 \\ 365 \\ \hline 104 \\ 20 \\ \hline 2087(5 \\ 1825 \\ \hline 262 \\ 12 \\ \hline 3144(8 \\ 2920 \\ \hline 224 \\ 4 \\ \hline 896(2 \\ 730 \end{array}$</p>
$\begin{array}{r} \text{£} \quad \text{s} \quad \text{d} \\ 22 \quad 7 \quad 0 \\ \hline 5 \\ 111 \quad 15 \quad 0 \\ 6 \text{ months} = 11 \quad 3 \quad 6 \\ 3 \text{ months} = 5 \quad 11 \quad 9 \\ 21 \text{ days} = 1 \quad 5 \quad 8\frac{1}{2} \\ \text{Principal} = 558 \quad 15 \quad 0 \\ \hline \text{Amount} = 688 \quad 10 \quad 11\frac{1}{2} \end{array}$	

(10)
From September 30th, 1839,
to June 18th, 1840, are 262
days.

$$\begin{array}{r}
 \text{£.} \\
 500 \\
 3\frac{1}{2} \\
 \hline
 1500 \\
 \frac{1}{2} = 250 \\
 \hline
 17,50 \\
 20 \\
 \hline
 10,00
 \end{array}$$

days. *days.* *£.* *s.*
As 365 : 262 :: 17 10
17 $\frac{1}{2}$

$$\begin{array}{r}
 1834 \\
 262 \\
 131 \\
 \hline
 365)4585
 \end{array}$$

(12l. 11s. 2 $\frac{1}{2}$ d. 67 days =

(11)

$$\begin{array}{r}
 \text{£.} \\
 450 \\
 3\frac{1}{2} \\
 \hline
 1350 \\
 \frac{1}{2} = 225 \\
 \frac{1}{2} = 112 \ 10 \\
 \hline
 16,87 \ 10 \\
 20 \\
 \hline
 17,50 \\
 12 \\
 \hline
 6,00
 \end{array}$$

£. *s.* *d.*
16 17 6 = interest
2 $\frac{1}{2}$ [for 1 yr.

$$\begin{array}{r}
 33 \ 15 \ 0 \\
 \frac{1}{2} = 8 \ 8 \ 9 \\
 \frac{1}{2} = 4 \ 4 \ 4\frac{1}{2} \\
 3 \ 1 \ 11\frac{1}{2} \\
 \hline
 49 \ 10 \ 0\frac{1}{2}
 \end{array}$$

Ans.

SIMPLE INTEREST BY DECIMALS.

(2)

$$\begin{array}{r}
 \text{£.} \\
 115 = \text{principal.} \\
 \cdot 045 = \text{rate.} \\
 \hline
 575 \\
 460 \\
 \hline
 5 \cdot 175 \\
 5 \cdot 5 = 5\frac{1}{2} \text{ years.} \\
 \hline
 25875 \\
 25875 \\
 \hline
 28,4625 \\
 20 \\
 \hline
 9,2500 \\
 12 \\
 \hline
 3,0000
 \end{array}$$

Ans. £28 9s. 3d.

(3)

$$\begin{array}{r}
 \text{£.} \\
 880 = \text{principal.} \\
 \cdot 035 = 3\frac{1}{2} \text{ per cent.} \\
 \hline
 4400 \\
 2640 \\
 \hline
 30 \cdot 800 \\
 1 \cdot 25 = 1\frac{1}{4} = \text{time.} \\
 \hline
 154000 \\
 369600 \\
 \hline
 38 \cdot 50000 \\
 20 \\
 \hline
 10 \cdot 00000 \text{ } \textit{Ans.} \text{ } £38 \ 10\text{s.}
 \end{array}$$

(4)	(5)
£	£
537.75 = prin.	555 = principal.
.03625 = $3\frac{1}{4}$ per cent.	.0475 = $4\frac{1}{2}$ = rate.
<u>268875</u>	<u>2775</u>
107550	3885
<u>822650</u>	<u>2220</u>
161325	26.3625
<u>19.4934375</u>	<u>2.5 = $2\frac{1}{2}$ years.</u>
206	1318125
<u>1169606250</u>	<u>527250</u>
389868750	65.90625
<u>365)40156481250</u>	<u>20</u>
11,0017756	18.12500
20	12
<u>0,0355120</u>	<u>1.50000</u>
12	4
<u>0,426144</u>	<u>2.00000</u>
4	
<u>1,704576</u>	
Ans. £11 0s. $0\frac{1}{2}$ d.	Ans. £65 18s. $1\frac{1}{2}$ d.

COMPOUND INTEREST, p. 146.

(2)	(2)	(2)
£. s.	£. s. d.	£. s. d.
15 10	15 10 0	16 0 10
<u>3½</u>	<u>10 10</u>	<u>0 11 2½</u>
46 10	16 0 10	16 12 0½
<u>½ = 7 15</u>	<u>3½</u>	<u>3½</u>
54 5	48 2 6	49 16 2½
<u>20</u>	<u>½ = 8 0 5</u>	<u>½ = 8 6 0½</u>
s. 10,85	56 2 11	58 2 2½
12	20	20
<u>d. 10,20</u>	s. 11,22	s. 11,62
	12	12
	d. 2,75	d. 7,46
	4	4
	<u>f. 3,00</u>	<u>f. 1,86</u>

Hence £16 12s. $0\frac{1}{2}$ d + 11s. $7\frac{1}{2}$ d = £17 3s. 8d. = interest req.

(4)

For quarterly payments,
divide by 80.

$$\begin{array}{r}
 20 \overline{) 50} \\
 \underline{2 \ 10} \\
 20 \overline{) 52 \ 10} = 2^{\text{nd}} \text{ principal.} \\
 \underline{2 \ 12 \ 6} = 2^{\text{nd}} \text{ interest.} \\
 20 \overline{) 55 \ 2 \ 6} = 3^{\text{rd}} \text{ principal.} \\
 \underline{2 \ 15 \ 1\frac{1}{2}} \\
 20 \overline{) 57 \ 17 \ 7\frac{1}{2}} \\
 \underline{2 \ 17 \ 10\frac{1}{2}} \\
 20 \overline{) 60 \ 15 \ 6} \\
 \underline{3 \ 0 \ 9\frac{1}{2}} \\
 \underline{63 \ 16 \ 3\frac{1}{2}} = \text{amount.}
 \end{array}$$

For half-yearly payments,
divide by 40.

$$\begin{array}{r}
 40 \overline{) 50} \\
 \underline{1 \ 5} \\
 40 \overline{) 51 \ 5} \\
 \underline{1 \ 5 \ 7\frac{1}{2}} \\
 40 \overline{) 52 \ 10 \ 7\frac{1}{2}} \\
 \underline{1 \ 6 \ 3} \\
 40 \overline{) 53 \ 16 \ 10\frac{1}{2}} \\
 \underline{1 \ 6 \ 11} \\
 40 \overline{) 55 \ 3 \ 9\frac{1}{2}} \\
 \underline{1 \ 7 \ 7} \\
 40 \overline{) 56 \ 11 \ 4\frac{1}{2}} \\
 \underline{1 \ 8 \ 3\frac{1}{2}} \\
 40 \overline{) 57 \ 19 \ 7\frac{1}{2}} \\
 \underline{1 \ 8 \ 11\frac{1}{2}} \\
 40 \overline{) 59 \ 8 \ 7\frac{1}{2}} \\
 \underline{1 \ 9 \ 8\frac{1}{2}} \\
 40 \overline{) 60 \ 18 \ 4} \\
 \underline{1 \ 10 \ 5\frac{1}{2}} \\
 40 \overline{) 62 \ 8 \ 9\frac{1}{2}} \\
 \underline{1 \ 11 \ 2\frac{1}{2}} \\
 \underline{64 \ 0 \ 0} = \text{amount.}
 \end{array}$$

$$\begin{array}{r}
 80 \overline{) 50} \\
 \underline{0 \ 12 \ 6} \\
 80 \overline{) 50 \ 12 \ 6} \\
 \underline{0 \ 12 \ 7\frac{1}{2}} \\
 80 \overline{) 51 \ 5 \ 1\frac{1}{2}} \\
 \underline{0 \ 12 \ 9\frac{1}{2}} \\
 80 \overline{) 51 \ 17 \ 11\frac{1}{2}} \\
 \underline{0 \ 12 \ 11\frac{1}{2}} \\
 80 \overline{) 52 \ 10 \ 11} \\
 \underline{0 \ 13 \ 1\frac{1}{2}} \\
 80 \overline{) 53 \ 4 \ 0\frac{1}{2}} \\
 \underline{0 \ 13 \ 3\frac{1}{2}} \\
 80 \overline{) 53 \ 17 \ 4} \\
 \underline{0 \ 13 \ 5\frac{1}{2}} \\
 80 \overline{) 54 \ 10 \ 9\frac{1}{2}} \\
 \underline{0 \ 13 \ 7\frac{1}{2}} \\
 80 \overline{) 55 \ 4 \ 5} \\
 \underline{0 \ 13 \ 9\frac{1}{2}} \\
 80 \overline{) 55 \ 18 \ 2\frac{1}{2}} \\
 \underline{0 \ 13 \ 11\frac{1}{2}} \\
 80 \overline{) 56 \ 12 \ 2} \\
 \underline{0 \ 14 \ 1\frac{1}{2}} \\
 80 \overline{) 57 \ 6 \ 3\frac{1}{2}} \\
 \underline{0 \ 14 \ 3\frac{1}{2}} \\
 80 \overline{) 58 \ 0 \ 7\frac{1}{2}} \\
 \underline{0 \ 14 \ 6} \\
 80 \overline{) 58 \ 15 \ 1\frac{1}{2}} \\
 \underline{0 \ 14 \ 8\frac{1}{2}} \\
 80 \overline{) 59 \ 9 \ 10} \\
 \underline{0 \ 14 \ 10\frac{1}{2}} \\
 80 \overline{) 60 \ 4 \ 8\frac{1}{2}} \\
 \underline{0 \ 15 \ 0\frac{1}{2}} \\
 80 \overline{) 60 \ 19 \ 8\frac{1}{2}} \\
 \underline{0 \ 15 \ 2\frac{1}{2}} \\
 80 \overline{) 61 \ 14 \ 11\frac{1}{2}}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 80)61 \quad 14 \quad 11\frac{1}{2} \\
 \underline{0 \quad 15 \quad 5\frac{1}{2}} \\
 80)62 \quad 10 \quad 4\frac{1}{2} \\
 \underline{0 \quad 15 \quad 7\frac{1}{2}} \\
 80)63 \quad 6 \quad 0\frac{1}{2} \\
 \underline{0 \quad 15 \quad 9\frac{1}{2}} \\
 64 \quad 1 \quad 10 = \text{amount.}
 \end{array}
 \end{array}$$

COMPOUND INTEREST BY DECIMALS.

(2)

$$\begin{array}{r}
 (1.04)^4 = 1.16985856 \\
 \text{Principal} = \underline{760.5} \\
 584929280 \\
 701915136 \\
 818900992 \\
 \underline{\text{£}889.677434880} \\
 20 \\
 \text{s. } 13.548697600 \\
 12 \\
 \text{d. } 6.584371200 \\
 4 \\
 \text{f. } \underline{2.337484800}
 \end{array}$$

(4)

$$\begin{array}{r}
 (1.04)^{21} = 2.278766 \\
 \underline{721 = \text{prin.}} \\
 2278766 \\
 4557532 \\
 15951362 \\
 1642.990286 \\
 20 \\
 19.805720 \\
 12 \\
 9.668640 \\
 4 \\
 \underline{2.674560}
 \end{array}$$

Ans. £1642 19s. 9½d.

(3)

By Ex. (2), we have

$$\begin{array}{r}
 \text{Amount} = \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 889 \quad 13 \quad 6\frac{1}{2} \end{array} \\
 \text{Principal} = \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 760 \quad 10 \quad 0 \end{array} \\
 \text{Interest} = \underline{\underline{\begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 129 \quad 3 \quad 6\frac{1}{2} \end{array}}}
 \end{array}$$

(5)

Here the amount of £1 for 1 quarter of a year = 1.0125
and the number of payments 9*.

Now $(1.0125)^9 = 1.1182921$

217 = principal.

$$\begin{array}{r}
 78280447 \\
 11182921 \\
 22365842 \\
 \hline
 242\cdot6693857 \\
 20 \\
 \hline
 13\cdot3877140 \\
 12 \\
 \hline
 4\cdot6525680 \\
 4 \\
 \hline
 2\cdot6102720 \quad \text{Ans. } £242 \text{ } 13s. \text{ } 4\frac{1}{2}d.
 \end{array}$$

DISCOUNT, p. 151.

(2)

As 101*l.* 5*s.* : 60*l.* :: 1*l.* 5*s.* : 14*s.* 10*d.* = discount.

(3)

As 113*l.* 10*s.* : 573*l.* 15*s.* :: 13*l.* 10*s.* : 68*l.* 4*s.* 10½*d.* =
discount.

(4)

As 12 mon. : 5 mon. :: 3*l.* 17*s.* 6*d.* : 1*l.* 12*s.* 3½*d.*

And 101*l.* 12*s.* 3½*d.* : 725*l.* 16*s.* :: 1*l.* 12*s.* 3½*d.* : 111*l.* 10*s.* 7½*d.*

(5)

As 110*l.* : 1000*l.* :: 10*l.* : 90*l.* 18*s.* 2*d.* = discount.

And $\frac{1000 \times 5}{100} = 50*l.*$; hence $\frac{100 \quad 0 \quad 0}{9 \quad 1 \quad 10} = \text{interest.}$

$\frac{100 \quad 0 \quad 0}{9 \quad 1 \quad 10} = \text{diff.}$

* If P = principal, r = interest of 1*l.* for 1 year, n = number of years; then amount at compound interest = $P \cdot (1+r)^n$.

If the interest is payable half-yearly, amount = $P \cdot (1+\frac{1}{2}r)^{2n}$.

If the interest is payable quarterly, amount = $P \cdot (1+\frac{1}{4}r)^{4n}$.

Whenever n is a great number, the process for obtaining $(1+r)^n$, or $(1+\frac{1}{2}r)^{2n}$, &c., is very tedious, and is best performed by Logarithms.

For additional information on this subject, see Tyson's Edition of "Bonycastle's Algebra."

(6)

As 106*l.* 5*s.* : 75*l.* :: 100*l.* : 70*l.* 11*s.* 9*d.* = present worth.

(7)

As 365 days : 66 days :: 5*l.* : 18 $\frac{2}{3}$ *s.* = interest of 5*l.* for 66 days; then

100*l.* 18 $\frac{2}{3}$ *s.* : 85*l.* 10*s.* :: 18 $\frac{2}{3}$ *s.* : 15*s.* 3 $\frac{1}{2}$ *d.* = discount.

(8)

	<i>£.</i>	<i>s.</i>	<i>£.</i>	<i>£.</i>	<i>£.</i>	<i>s.</i>	<i>d.</i>					
As 101	10		:	50	:	:	100	:	49	5	2 $\frac{1}{2}$	= present worth.
102	10		:	50	:	:	100	:	48	15	7 $\frac{1}{2}$	= present worth.
104	0		:	20	:	:	100	:	19	4	7 $\frac{1}{2}$	= present worth.
									<u>117</u>	<u>5</u>	<u>5</u>	= whole present worth.

DISCOUNT BY DECIMALS.

(2)

Here the rate = 4·375; time = 2 years, 3 qrs. 25 days = 2·818493 years; $\therefore 2\cdot818493 \times 4\cdot375 = 12\cdot330906875$, which, being divided by 100, according to the rule for interest, will give $\cdot12330906875$ = the interest of 1*l.* for 2 years, 3 qrs. 25 days. Hence $1\cdot12330906875$ = amount of 1*l.* at 4 $\frac{3}{4}$ per cent. during 2 years, 3 qrs. 25 days. Therefore

1·1233090685 : 1*l.* :: 1377·666 : 1226*l.* 8*s.* 8 $\frac{1}{2}$ *d.*,
the present worth required.

This question may be solved thus: first find the interest of 1377*l.* 13*s.* 4*d.* for 2 $\frac{3}{4}$ years, 25 days; then say,

As amount of 1*l.* for given rate and time as found above : 1*l.* :: the interest of the debt : the discount required.

Subtract the discount from the debt, and the remainder will be the present worth.

(4)

$$\begin{array}{rcl}
 100 + 100 & = & 200 \\
 100 \times .045 \times 1 & = & 4.5 \\
 \text{1st No.} & = & 204.5 \\
 \hline
 \text{Square of 1st No.} & = & 41820.25 \\
 4 \times 100^2 \times .045 \times 1 & = & 1800 \\
 \hline
 \text{Diff.} & = & 40020.25
 \end{array}
 \quad
 \begin{array}{l}
 \therefore \sqrt{40020.25} = 200.05 \\
 \text{1st No.} = 204.5 \\
 \text{Diff.} = 4.45 \\
 2 \times .045 \times 100 = 9, \text{ and} \\
 4.45 \div 9 = .4944 \text{ yr., and} \\
 2.5 + .4944 = 2.9944 \text{ yrs.} \\
 = \text{equated time.}
 \end{array}$$

(5)

$$\begin{array}{rcl}
 \text{Sum of the two payments} & = & 420\text{£} \\
 \text{First payment} \times \text{ratio} \times \text{interval} & = & 9.84375 \\
 \hline
 \text{1st No.} & = & 429.84375 \\
 \hline
 \text{Square of 1st No.} & = & 184765.6494140625 \\
 4 \times 157.5 \times 262.5 \times 1.25 \times .05 & = & 10335.9375 \\
 \hline
 & = & 174429.7119140625 \\
 \text{The square root of this number} & = & 417.648 = \text{2nd No.} \\
 & & 429.843 = \text{1st No.} \\
 & & \hline
 & & 12.195 = \text{the diff.} \\
 2 \times .05 \times 157.5 & = & 15.75 \\
 \hline
 \text{Hence } \frac{12.195}{15.75} & = & .774; \text{ and the equated time required} = \\
 1\frac{1}{2} + .774 & = & 2.524 \text{ years from the present time.}
 \end{array}$$

(6)

$$\begin{array}{rcl}
 100 + 100 & = & 200 \\
 100 \times .06 \times 2 & = & 12 \\
 \hline
 212 \times 212 & = & 44944 = \text{square of 1st No.} \\
 4 \times 100 \times 100 \times .06 \times 2 & = & 4800 \\
 \hline
 & = & 40144 \\
 \text{Now } \sqrt{40144} & = & 200.359 = \text{2nd No.} \\
 & & 212 = \text{1st No.} \\
 & & \hline
 & & 11.641 = \text{diff.} \\
 \text{But } 2 \times .06 \times 100 & = & 12, \text{ and } 11.641 \div 12 = .97, \text{ and the} \\
 \text{equated time} & = & 2.97 \text{ years.}
 \end{array}$$

ANNUITIES, p. 166.

PROBLEM I.

(2)	(3)
Interest of 600 <i>l.</i> = $\begin{array}{r} \text{£} \\ 24 \\ 1+2+3+4 = 10 \\ \hline \text{Total interest} = 240 \\ 600 \times 5 = 3000 \\ \hline \text{Amount} = \underline{3240} \end{array}$	Interest of 125 <i>l.</i> = $\begin{array}{r} \text{£} \\ 3\cdot75 \\ 1+2+3+\dots 13 = 91 \\ \hline \text{Total interest} = 341\cdot25 \\ 125 \times 14 = 1750 \\ \hline \text{Amount} = \underline{2091\text{ }l. 5s.} \end{array}$

(4)	(5)
Interest of 100 <i>l.</i> = $\begin{array}{r} \text{£} \\ 4\cdot5 \\ 1+2+3+\dots 6 = 21 \\ \hline \text{Total interest} = 94\cdot5 \\ 100 \times 7 = 700 \\ \hline \text{Amount} = \underline{794\text{ }l. 10s.} \end{array}$	Interest of 35 <i>l.</i> = $\begin{array}{r} \text{£} \\ 8\cdot75 \\ 1+2+3+\dots 9 = 45 \\ \hline \text{Total interest} = 39\cdot375 \\ 70 \times 5 = 350 \\ \hline \text{Amount} = \underline{389\text{ }l. 7s. 6d.} \end{array}$

(6)
Interest of 17 <i>l.</i> 10 <i>s.</i> for $\frac{1}{2}$ year = $\begin{array}{r} \text{£} \\ 218\cdot75 \\ 1+2+3+4+\dots 19 = 190 \\ \hline \text{Total interest} = 41\cdot5625 \\ 17\text{ }l. 10s. \times 20, \text{ or } 70 \times 5 = 350 \\ \hline \text{Amount} = \underline{391\cdot5625} = 391\text{ }l. 11s. 3d. \end{array}$

PROBLEM II.

(2)

As 105 : 100 :: 500 : 476·19047
110 : 100 :: 500 : 454·54545
115 : 100 :: 500 : 434·78260
120 : 100 :: 500 : 416·66666
Sum of the present worths = $\begin{array}{r} 1782\cdot18518 \\ \hline = \underline{1782\text{ }l. 3s. 8\frac{1}{2}d.} \end{array}$

PROBLEM III.

(2)

$$\begin{array}{rcl}
 1 & = & 1 \\
 1.04 & = & 1.04 \\
 (1.04)^2 & = & 1.0816 \\
 (1.04)^3 & = & 1.124864 \\
 (1.04)^4 & = & 1.169858 \\
 (1.04)^5 & = & 1.216653 \\
 (1.04)^6 & = & 1.265320 \\
 & & \underline{7.898295} \\
 & & 50 \\
 \text{Amount} & = & 394.914750 \\
 & = & \underline{394\text{ l. } 18\text{ s. } 3\frac{1}{2}\text{ d.}}
 \end{array}$$

(3)

In this example the payments are made quarterly; also $1.0125 =$ amount of 1*l.* for one quarter at 5 per cent.

$$\begin{array}{rcl}
 1 & = & 1 \\
 1.0125 & = & 1.0125 \\
 (1.0125)^2 & = & 1.02516 \\
 (1.0125)^3 & = & 1.03797 \\
 (1.0125)^4 & = & 1.05094 \\
 (1.0125)^5 & = & 1.06407 \\
 (1.0125)^6 & = & 1.07737 \\
 & & \underline{7.26801} \\
 & & 18.75 = \text{one quar-} \\
 & & \underline{136.2751875} \text{ [ter's paymt.]} \\
 \text{Amount} & = & \underline{136\text{ l. } 5\text{ s. } 6\text{ d.}}
 \end{array}$$

(4)

Here $1.055 =$ amount of 1*l.* for one year at $5\frac{1}{2}$ per cent.

$$\begin{array}{rcl}
 \text{Also } 1 & = & 1 \\
 1.055 & = & 1.055 \\
 (1.055)^2 & = & 1.11302 \\
 (1.055)^3 & = & 1.17424 \\
 (1.055)^4 & = & 1.23882 \\
 (1.055)^5 & = & 1.30695 \\
 & & \underline{6.88803} \\
 & & 35 \\
 \text{Amount} & = & 241.08105 \\
 & = & \underline{241\text{ l. } 1\text{ s. } 7\frac{1}{2}\text{ d.}}
 \end{array}$$

(5)

Here $1.0225 =$ amount of 1*l.* for a half-year at $4\frac{1}{2}$, and the number of payments = 7.

$$\begin{array}{rcl}
 \text{Now } 1 & = & 1 \\
 1.0225 & = & 1.0225 \\
 (1.0225)^2 & = & 1.04550 \\
 (1.0225)^3 & = & 1.06902 \\
 (1.0225)^4 & = & 1.09307 \\
 (1.0225)^5 & = & 1.11766 \\
 (1.0225)^6 & = & 1.14280 \\
 & & \underline{7.49055} \\
 \frac{1}{2} \text{ yearly payment} & = & 49.25 \\
 \text{Amount} & = & 368.90958 \\
 & = & \underline{368\text{ l. } 18\text{ s. } 2\frac{1}{2}\text{ d.}}
 \end{array}$$

PROBLEM IV.

(2)	(3)
Now $21l. 10s. 9\frac{1}{2}d. = 21.5395833l.$	$\frac{70}{1.05} = 66.66666$
Also $\frac{21.5395833}{1.06} = 20.3203$	$\frac{70}{(1.05)^2} = 63.49206$
$\frac{21.5395833^*}{(1.06)^2} = 19.1701$	$\frac{70}{(1.05)^3} = 60.46863$
$\frac{21.5395833}{(1.06)^3} = 18.0850$	$\frac{70}{(1.05)^4} = 57.58917$
$\frac{21.5395833}{(1.06)^4} = 17.0613$	$\frac{70}{(1.05)^5} = 54.84683$
$\frac{21.5395833}{(1.06)^5} = 16.0955$	Present value = 303.06335
$\frac{21.5395833}{(1.06)^6} = 15.1845$	$= 303l. 1s. 3\frac{1}{2}d.$
$\frac{21.5395833}{(1.06)^7} = 14.3250$	
Present value = 120.2417	
$= 120l. 4s. 10d.$	

(4)

By the note at p. 161 of the Arithmetic, we have

$$p = \frac{r^n - 1}{r^n(r-1)} a = \frac{1 - \frac{1}{r^n}}{r-1} a,$$

$$\text{or } a = \frac{p(r-1)}{1 - \frac{1}{r^n}} = \frac{p(r-1)}{1 - r^{-n}}.$$

$$\therefore \log a = \log p + \log(r-1) - \log(1 - r^{-n}).$$

Now $p = 100l.$, $r = 1.055l.$, and $n = 55.25$ years;

$$\begin{aligned} \therefore \log r^{-n} &= -n \log r = -55.25 \times .0232525 \\ &= -1.284700625 = \bar{2}.7152994 \\ &= \log .05191577; \therefore r^{-n} = .05191577. \end{aligned}$$

* It is not necessary to expand the denominators, for each succeeding quotient is obtained by dividing the preceding one by 1.06.

Hence $1 - r^{-n} = 1 - \cdot 05191577 = \cdot 94808423$;

$$\therefore \log p = \log 100 = 2\cdot 0000000$$

$$\log (r-1) = \log \cdot 055 = 2\cdot 7403627$$

$$\underline{0\cdot 7403627}$$

$$\log (1 - r^{-n}) = \log \cdot 94808423 = 1\cdot 9768450$$

$$\log a = 0\cdot 7635177$$

$$a = 5\cdot 80111\text{ l.} = 5\text{ l. } 16\text{ s.} = \text{annuity.}$$

	(5)
$\frac{126}{1\cdot 05} = 120\cdot$	Sum bt. forward 1046·59
$\frac{126}{(1\cdot 05)^2} = 114\cdot 28$	$\frac{126}{(1\cdot 05)^{12}} = 70\cdot 17$
$\frac{126}{(1\cdot 05)^3} = 108\cdot 84$	$\frac{126}{(1\cdot 05)^{13}} = 66\cdot 83$
$\frac{126}{(1\cdot 05)^4} = 103\cdot 66$	$\frac{126}{(1\cdot 05)^{14}} = 63\cdot 64$
$\frac{126}{(1\cdot 05)^5} = 98\cdot 72$	$\frac{126}{(1\cdot 05)^{15}} = 60\cdot 61$
$\frac{126}{(1\cdot 05)^6} = 94\cdot 02$	$\frac{126}{(1\cdot 05)^{16}} = 57\cdot 73$
$\frac{126}{(1\cdot 05)^7} = 89\cdot 55$	$\frac{126}{(1\cdot 05)^{17}} = 54\cdot 97$
$\frac{126}{(1\cdot 05)^8} = 85\cdot 28$	$\frac{126}{(1\cdot 05)^{18}} = 52\cdot 35$
$\frac{126}{(1\cdot 05)^9} = 81\cdot 22$	$\frac{126}{(1\cdot 05)^{19}} = 49\cdot 86$
$\frac{126}{(1\cdot 05)^{10}} = 77\cdot 35$	$\frac{126}{(1\cdot 05)^{20}} = 47\cdot 48$
$\frac{126}{(1\cdot 05)^{11}} = 73\cdot 67$	$\frac{126}{(1\cdot 05)^{21}} = 45\cdot 23$
<u>1046·59</u>	Present value = <u>1615·46</u>
	<u>= 1615 l. 9 s. 3 d.</u>

PROBLEM V.

(2)

$$\text{As } 4\cdot 5 : 100 :: 79\text{ l. } 4\text{ s.} : \frac{79\cdot 2 \times 100}{4\cdot 5} = 1760\text{ l.}$$

(3)

$$\text{As } 5 : 100 :: 40 : \frac{40 \times 100}{5} = 800\text{L.}$$

(4)

As $5 : 100 :: 100 : 2000\text{L.}$ = value of the perpetuity of 100L. per annum.

Next, to find the value of the 60 years' lease, we have

$$p = \frac{1 - \frac{1}{r^n}}{r - 1} \cdot a = \frac{1 - r^{-n}}{r - 1} a;$$

$$\therefore \log p = \log a + \log (1 - r^{-n}) - \log (r - 1).$$

$$\log 1 \cdot 05^{-60} = -60 \times \cdot 0211893 = -1 \cdot 2713580$$

$$= \bar{2} \cdot 7286420 = \log \cdot 0535355;$$

$$\therefore 1 \cdot 05^{-60} = \cdot 0535355, \text{ and hence}$$

$$1 - 1 \cdot 05^{-60} = 1 - \cdot 0535355 = \cdot 9464645.$$

$$\text{Now } \log a = \log 100 = 2 \cdot 0000000$$

$$\log (1 - r^{-n}) = \log \cdot 9464645 = \bar{1} \cdot 9761043$$

$$\underline{1 \cdot 9761043}$$

$$\log (r - 1) = \log \cdot 05 = \bar{2} \cdot 6989700$$

$$\log p = \underline{\underline{3 \cdot 2771343}}$$

$$\therefore p = 1892 \cdot 929\text{L.} = \begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 1892 \quad 18 \quad 6\frac{1}{2} \end{array}$$

$$\text{Perpetuity} = \begin{array}{r} 2000 \quad 0 \quad 0 \end{array}$$

$$\text{Difference} = \underline{\underline{\text{£}107 \quad 1 \quad 5\frac{1}{2}}}$$

(5)

$$\text{As } 4 : 100 :: 150\text{L.} : \frac{15000}{4} = 3750\text{L.}$$

(6)

$$\text{As } 3 : 100 :: 150\text{L.} : \frac{15000}{3} = 5000\text{L.}$$

(7)

$$\text{As } 5 : 100 :: 150\text{L.} : \frac{15000}{5} = 3000\text{L.}$$

PROBLEM VI.

(2)

$$\frac{1}{1.05} = .952381$$

$$\frac{1}{(1.05)^2} = .907029$$

$$\frac{1}{(1.05)^3} = .863838$$

$$\frac{1}{(1.05)^4} = .822702$$

$$\frac{1}{(1.05)^5} = .783526$$

$$\frac{1}{(1.05)^6} = .746215$$

$$\frac{1}{(1.05)^7} = .710681^* \quad 5.786372$$

$$\frac{1}{(1.05)^8} = .676839$$

$$\frac{1}{(1.05)^9} = .644609$$

$$\frac{1}{(1.05)^{10}} = .613913$$

$$\frac{1}{(1.05)^{11}} = .584679$$

$$\frac{1}{(1.05)^{12}} = .556837$$

$$\frac{1}{(1.05)^{13}} = .530321$$

$$\frac{1}{(1.05)^{14}} = .505068$$

$$\frac{1}{(1.05)^{15}} = .481017$$

$$\frac{1}{(1.05)^{16}} = .458112$$

$$\frac{1}{(1.05)^{17}} = .432697$$

$$\frac{1}{(1.05)^{18}} = .415521$$

$$\frac{1}{(1.05)^{19}} = .395734$$

$$\frac{1}{(1.05)^{20}} = .376889$$

$$\frac{1}{(1.05)^{21}} = .358942$$

Present value for 21 years = 12.821150

Present value for 7 years = 5.786372

Value required = 7.034778

= 7l. 0s. 8½d.

(3)

This question is solved by the same process as that used in the last example. The annuity in this example being 100 times as great as the annuity in Ex. 1; therefore the value must be 100 times as great, and

$$7.034778 \times 100 = 703.4778\text{l.} = 703\text{l. } 8\text{s. } 9\text{d.}$$

* The sum of the column of figures above this line = the present value of 1l. per annum for 7 years.

(4)

$\log p = \log (1 - r^{-n}) - \log (r - 1)$; (a being = 1*l*.)

$\log r^{-n} = -n \log r = -20 \times \log 1.04 = -.340660$

$= \bar{1}.659340 = \log .4564$

$r^{-n} = .4564$ and $1 - r^{-n} = 1 - .4564 = .5436$

$\log (1 - r^{-n}) = \log .5436 = \bar{1}.735279$

$\log (r - 1) = \log .04 = \bar{2}.602060$

$\log p = \bar{1}.133219$

And $p = 13.59$ = present value of 1*l*. per ann. for 20 years.
Similarly, 11.11 = present value of 1*l*. per ann. for 15 years.

2.48 = present value of 1*l*. per annum during the
5 years of the running lease.

Hence $2.48 \times 186.375 = 462*l*. 4*s*. 2½*d*.$ = value of the
favour.

(5)

The estate is said to be worth 20*l*. per annum, but this has nothing to do with the solution of the problem, neither is it one of the necessary data. We have to find what annuity, to be continued 21 years, may be purchased with 100*l*., allowing 5 per cent. interest.

Since $p = a \cdot \frac{1 - r^{-n}}{r - 1}$, we have $a = \frac{p(r - 1)}{1 - r^{-n}}$;

$\therefore \log a = \log p + \log (r - 1) - \log (1 - r^{-n})$
 $= \log 100 + \log (.05) - \log \left(1 - \frac{1}{(1.05)^{21}} \right)^*$
 $= 2 + \bar{2}.698970 - \bar{1}.806858 = .892112.$

$\therefore a = 7.80*l*. = 7*l*. 16*s*.$ = the sum by which the rent ought
to be increased.

PROBLEM VII.

(2)

To find the value of the perpetuity, we have

$5 : 100 :: 100 : 2000*l*.$ = the value of the estate.

* The method of determining the values of expressions like this has been already given in the preceding pages.

$$\begin{aligned}
 \text{Now } \log p &= \log \left(1 - \frac{1}{r^n} \right) - \log (r-1) + \log a. \\
 &= \log \left(1 - \frac{1}{1.05^{15}} \right) - \log .05 + 2. \\
 &= 3.016182.
 \end{aligned}$$

$\therefore p = 1037.96$ = present value of the first term of 15 years.

$\frac{2000}{.05}$ = present value of the perpetuity.

962.04 = value of the reversion in fee; which value is less than 1037.96 by 75.92, or by 75*l.* 18*s.* nearly.

(3)

As 5 : 100 :: 91*l.* : 1820*l.* = value of the annuity for ever from the present time.

$$\begin{aligned}
 \text{Also } \log p &= \log a + \log \left(1 - \frac{1}{r^n} \right) - \log (r-1) \\
 &= \log 91 + \log \left(1 - \frac{1}{(1.05)^{17}} \right) - \log (.05) \\
 &= 1.959041 + 1.751048 - 2.698970 \\
 &= 3.011119
 \end{aligned}$$

$\therefore p = 1025.93$ = present value of 91*l.* per ann. for 17 years.

$\frac{1820}{.05}$ = value of the annuity for ever.

$\pounds 794.07$ = value of the reversion.

(4)

As 5 : 100 :: 20*l.* : 400*l.* = value of the perpetuity from the present time.

$$\text{Also } \frac{20}{1.05} = 19.04762$$

$$\frac{20}{(1.05)^2} = 18.14058$$

[be continued 2 years.

37.18820 = present value of 20*l.* per annum to

But 400 = value of perpetuity.

362.8118 = 362*l.* 16*s.* 2*d.* = present worth.

EXCHANGE.

ENGLAND WITH HOLLAND AND FLANDERS.

To reduce the currency of any state into banco or exchange money.

(2)

$$\text{As } 100 + 6.5 : 100 :: \overset{\text{fl. cur.}}{3000} : \overset{\text{fl. ban. stiv.}}{2816 \text{ 18.}}$$

To reduce banco money to currency.

(2)

$$\text{As } 100 : 108 :: \overset{\text{fl. ban.}}{5000} : \overset{\text{fl. cur.}}{5400.}$$

EXAMPLES IN EXCHANGE WITH HOLLAND.

(2)

<i>flor. stiv.</i>	<i>£.</i>	<i>flor. stiv. pen.</i>
As 12 1½ : 1 ::		4029 9 9
20		20
241 4 pen.		80589
16		16
1450		483543
241		80589
3860 pen.	3860)	1289433 pen. <u>(334l. 1s.</u>

(3)

<i>flor. stiv.</i>	<i>£.</i>	<i>flor. stiv.</i>
As 12 1½ : 1 ::		688 7
20		20
241 8 pen.		13767
16		16
1454		82602
241		13767
3864	3864)	220272 (57l. 0s. 1½d. Ans.

(4)

$$\text{As } \overset{\text{flor. stiv.}}{12} \ 2 : \overset{\text{flor.}}{3630} :: \overset{\text{£.}}{1} : \overset{\text{£.}}{300} \text{ Ans.}$$

HAMBURG AND ALTONA.

$$\begin{array}{ccccccc} & & & (2) & & & \\ \text{As } 1 : 300 & :: & 11 & 10 : 3487 & 8 & \text{Ans.} \end{array}$$

FRANCE.

$$\begin{array}{ccccccc} & & & (2) & & & \\ \text{As } 24 : 16914 & 19 : : & 1 : 704 & 15 & 1\frac{1}{2}, \end{array}$$

$$\begin{array}{ccccccc} & & & (3) & & & \\ \text{As } 25 & 60 : 10981 & 62 : : & 1 : 428 & 19 & 4\frac{1}{2}. \end{array}$$

$$\begin{array}{ccccccc} & & & (4) & & & \\ \text{As } 1 : 213 & 2 : : & 25 & 47\frac{1}{2} : 5428 & 72 & \text{Ans.} \end{array}$$

SPAIN.

$$\begin{array}{l} (2) \\ \text{Now a real of plate : a real of vellon} :: 32 : 17; \\ \therefore 32 : 17 :: 5647 \text{ reals of vellon.} \end{array}$$

$$\begin{array}{r} 17 \\ 39529 \\ 5647 \\ \hline 32)95999(2999 \text{ reals, } 15\frac{1}{2} \text{ quartos. } \text{Ans.} \end{array}$$

$$\begin{array}{ccccccc} & & & (3) & & & \\ \text{As } 33\frac{1}{2} : 100 & :: & 1 : 716 & 3 & 11\frac{1}{2}. \end{array}$$

$$\begin{array}{r} (4) \\ \text{ducats. } 808 \quad \text{mar. } 222 \\ 375 \\ \hline 34)303222 \text{ maravedies.} \\ 8)8918 \text{ reals, } 10 \text{ mar.} \\ \hline 1114 \text{ pesos, } 6 \text{ reals, } 10 \text{ mar.} \end{array}$$

$$\begin{array}{ccccccc} \text{peso. } & \text{pesos. } & \text{ris. } & \text{mar. } & \text{d. } & \text{£. } & \text{s. } & \text{d. } \\ \text{Also } 1 : 1114 & 6 & 10 : : & 35\frac{1}{2} : 166 & 1 & 1\frac{1}{2}. \end{array}$$

PORTUGAL.

(1)

$$\text{As } 64\frac{1}{2} : 750 :: 1000 : 2785299.$$

(2)

$$\text{As } 64\frac{1}{2} : 754 \text{ } 18 \text{ } 6 :: 1000 : 2809023 = 7022\frac{1}{2}\frac{1}{2}.$$

(3)

$$\text{mil. mil. reas. d. } \text{As } 1 : 827 \text{ } 160 :: 63\frac{1}{2} : 218 \text{ } 8 \text{ } 5\frac{1}{2}.$$

(4)

$$\text{mil. crusa. reas. s. d. } \text{As } 1 : 2729 \text{ } 372 :: 5 \text{ } 2 : 282 \text{ } 1 \text{ } 10\frac{1}{2}.$$

RUSSIA.

(2)

$$\text{rubl. rubles. d. } \text{As } 1 : 7343 :: 11\frac{1}{2} : 359 \text{ } 10 \text{ } 0\frac{1}{2}.$$

(3)

$$\text{d. } \text{As } 12\frac{1}{2} : 322 \text{ } 10 \text{ } 6 :: 1 : 6318 \text{ } 85\frac{1}{2}\frac{1}{2}.$$

(4)

$$\text{d. } \text{As } 12\frac{1}{2} : 500 \text{ } 1 \text{ } 4 :: 1 : 9413 \text{ } 1\frac{1}{2}\frac{1}{2}.$$

SICILY.

(2)

$$\text{d. } \text{As } 125 : 566 \text{ } 10 \text{ } 6 :: 1 : 1087 \text{ } 21 \text{ } 16\frac{1}{2}.$$

(3)

$$\text{taris. as. taris. } \text{As } 60 : 470 \text{ } 6 :: 1 : 235 \text{ } 2.$$

NAPLES.

(2)

$$\text{d. } \text{As } 39\frac{1}{2} : 351 \text{ } 0 \text{ } 9 :: 1 : 2132 \text{ } 89.$$

(3)

$$\text{grs. ducats. } \text{As } 601 : 2560 :: 1 : 425 \text{ } 19 \text{ } 1.$$

TRIESTE, VIENNA, AND PRAGUE.

(2)

	<i>flor. kr.</i>	<i>flor.</i>	<i>kr.</i>	<i>£.</i>	<i>£.</i>	<i>s.</i>	<i>d.</i>
As 10	8	560	22	:	1	55	5 11½.

(3)

	<i>£.</i>	<i>£.</i>	<i>s.</i>	<i>d.</i>	<i>flor. kr.</i>	<i>flor.</i>	<i>kr.</i>
As 1	:	73	6	6	:	9	50 : 721 1½.

(4)

	<i>flor. kr.</i>	<i>flor.</i>	<i>kr.</i>	<i>£.</i>	<i>£.</i>	<i>s.</i>
As 9	54	:	547	28	:	1 : 55 6.

GENOA.

(2)

	<i>£.</i>	<i>£.</i>	<i>s.</i>	<i>d.</i>	<i>lire. cents.</i>	<i>li. nuo. cents.</i>
As 1	:	352	10	6	:	25 35 : 8936 50.

(3)

	<i>£.</i>	<i>£.</i>	<i>s.</i>	<i>lire. cents.</i>	<i>li. nuo. cents.</i>
As 1	:	230	6	:	25 40 : 5849 62.

LEGHORN.

(2)

	<i>pesse.</i>	<i>pesse.</i>	<i>soldi.</i>	<i>d.</i>	<i>£.</i>	<i>s.</i>	<i>d.</i>
As 1	:	6390	15	:	49½	:	1311 8 8½.

CONSTANTINOPLE AND SMYRNA.

(2)

	<i>pias.</i>	<i>pias.</i>	<i>cents.</i>	<i>£.</i>	<i>£.</i>	<i>s.</i>	<i>d.</i>
As 59½	:	14778	33	:	1	248	7 6.

(3)

	<i>£.</i>	<i>£.</i>	<i>s.</i>	<i>pias.</i>	<i>pias.</i>
As 1	:	355	10	:	60 : 21330.

(4)

	<i>£.</i>	<i>£.</i>	<i>s.</i>	<i>pias. cents.</i>	<i>pias. cents.</i>
As 1	:	220	10	:	62 50 : 13781 25.

BRITISH AMERICA AND WEST INDIES.

(2)

	<i>£.</i>	<i>£.</i>	<i>s.</i>	<i>d.</i>	<i>£.</i>	<i>s.</i>	<i>d.</i>	<i>£.</i>	<i>s.</i>	<i>d.</i>
As 100	:	133	6	8	:	576	19	6	:	767 19 4.

(3)

As 100	:	140	::	945	17			
or 5	:	7	::	945	17	:	1324	3 9½.

ARBITRATION OF EXCHANGES.

SIMPLE ARBITRATION.

(2)
 As 1*l.* or 240*d.* : 417*d.* (Flemish) :: 65½*d.*
 $\therefore \frac{417 \times 525}{240 \times 8} = 114\frac{1}{8}$ *d.* Flemish per millrea.
 Now a millrea : a crusado :: 10 : 4 :: 5 : 2.*
 $\therefore 5 : 2 :: 114\frac{1}{8} : 45\frac{3}{4}$ *d.* per crusado. *Ans.*

(3)
 As 1*l.* or 240*d.* : 429*d.* (Flemish) :: 32*d.* : 57½*d.*

ARITHMETICAL RECREATIONS.

(1)
 Since 2 pears cost 1½*d.* \therefore 1 pear costs ¾*d.*, and 17 pears or 12 apples cost 12¾*d.* or 12½*d.*; and 1 apple costs 1¼*d.*; \therefore 99 apples cost 99¼*d.* or 105¼*d.* or 8*s.* 9¼*d.* *Ans.*

(2)
 The even digits = 2 + 4 + 6 + 8 = 12·8;
 odd digits = 1 + 3 + 7 + 9 = 12·8.

(3) *
 One third part of 2*d.* = ⅔*d.* = ⅔ of 1 penny = ⅔ of ¼ of 3*d.* = ⅔ of 3*d.* *Ans.* ⅔.

(5)
 Three 5's placed thus, 5½, signify 6; and ·5 signifies ½; hence the four 5's ought to be placed thus, 5½·5.

(6)
 The arrangement of the figures is as follows:

8	3	4
1	5	9
6	7	2.

(7)
 I must give my friend 13 guineas, and he must give me 16 coins worth 17*s.* each; he will then have received 1*s.*

* Because 400 reas make one crusado, and 1000 reas make one millrea, in Portuguese monies.

(8)

There is wine sufficient to fill $10\frac{1}{2}$ bottles; hence, each person must have $3\frac{1}{2}$ bottles of wine and 7 bottles in all. The first person may therefore have 3 full bottles, one $\frac{1}{2}$ bottle, and 3 empty ones. The second may have 3 full bottles, one $\frac{1}{2}$ bottle, and 3 empty ones; and the third person, 1 full bottle, five $\frac{1}{2}$ bottles, and one empty.

(9)

Suppose there are just as many persons in the world as any one of them has hairs upon his head, then it is evident that the number of hairs on the head of each person may differ from the number on the head of any other person. But if there be *more* persons by one in the world than any one of them has hairs upon his head, this individual must have just as many hairs on his head as are on the head of some individual previously existing.

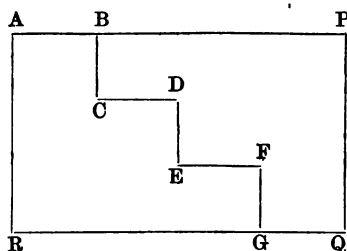
(10)

In this question it appears that 144 shillings were spent; and the number of shillings spent by each person was equal to the number of persons in the party.

Now $12 \times 12 = 144$, \therefore there were 12 persons.

(11)

The board ought to be 16 inches long and 9 broad; then by cutting it as represented by the following figure, the two parts joined together will form a square whose side = 12 inches.



Let $APQR$ be the board, $AP = 16$ inches, and $AR = 9$; take $AB = 4$ inches, draw BC parallel to AR , and let $BC = 3$ inches, $CD = 4$, $DE = 3$, $EF = 4$, $FG = 3$; then by cutting the board along the lines BC , CD , DE , EF , FG , and placing the point C at A , D will fall on B , E on C , &c., and Q on F ;

then R A will be lengthened 3 inches, and so become = 12 inches; R Q will be shortened 4 inches, and become = 12 inches; hence the figure thus formed will be a square.

(12)

Ans. 3 pigeons at 4*d.* each, cost . 12*d.*
 15 larks at $\frac{1}{2}$ *d.* each, cost . . 7 $\frac{1}{2}$
 2 sparrows at $\frac{1}{4}$ *d.* each, cost . $\frac{1}{2}$
 20 fowls cost 20

(13)

He must first fill the 3 gallon cask, and pour its content into the 5 gallon measure; then fill the 3 gallon cask again, and pour as much out of it into the 5 gallon cask as will just fill it; he will then have 1 gallon in the 3 gallon measure. The 5 gallon cask must then be emptied into the 8 gallon ditto, and the single gallon out of the 3 gallon measure into the 5 gallon ditto; then having filled the 3 gallon measure again, empty it into the 5 gallon measure, and there will then be 4 gallons in each of the two greater vessels.

(14)

The woman had 25 eggs.

(15)

Ans. 3 pigs, 81 sheep, and 16 geese.

(16)

There being 3 prsons, each peerson must have 16 lbs. of sugar, which may be weighed at 4 times with the 4 lb. weight alone. If there were 45 lbs of sugar, put the 4 lb. weight into one scale and the 7 lb. weight into the other, and put sufficient sugar to the 4 lb. weight to make it balance the 7 lb. weight; then there will be 3 lbs. of sugar in the scale. Now each person's share is 15 lbs.; hence three 4 lbs. weight of sugar and the 3 lbs. just mentioned will be one person's share, and the remainder must be equally divided.

(17)

$384 \div 32 = 12 =$ the number of ranks.

(18)

cwt. *lbs.* *lb.* *£.* *s.* *d.*
 As $1\frac{1}{2}$, or 168 : 1 : : 96 : 11 5 $\frac{1}{2}$.

Now the clear price for which the tea was sold = 117*l.*;

o 2

\therefore the nominal value received = 117*l.* + discount of 117*l.* for three months at 5 per cent. = 117*l.* + 1*l.* 8*s.* 10½*d.* = 118*l.* 1*s.* 10½*d.*; hence

$$\begin{array}{ccccccc} \text{lb.} & \text{lb.} & \text{£.} & \text{s.} & \text{d.} & \text{s.} & \text{d.} \\ 168 : 1 :: 118 & 8 & 10\frac{1}{2}\text{d.} : 14 & 1.* \end{array}$$

(19)

Now by selling 4 qrs. he gained $\frac{1}{3}$ of a qr. or $\frac{5}{37}$ of the quantity sold; hence, that which cost $\frac{2}{37}$, he sold for $\frac{27}{37}$; hence

$$37 : 32 :: \begin{array}{ccc} \text{£.} & \text{s.} & \text{d.} \\ 892 & 10 & 771 \end{array} : \begin{array}{ccc} \text{£.} & \text{s.} & \text{d.} \\ 771 & 17 & 10\frac{5}{7} \end{array} = \text{cost price.}$$

(20)

The minute hand travels 12 times as fast as the hour hand, and both travel uniformly; also the minute hand gains 11 spaces out of 12 upon the hour hand; hence

$$11 : 1 :: 12 : \frac{12}{11} = 1 \text{ hour, } 5\frac{5}{11} \text{ min.} = \text{Ans.}$$

(21)

Since C travels twice as fast as A, it is evident they can never come together except at the end of each complete circuit made by A: we have therefore to determine when A and B will be together in the same place. Now B gains 3 miles per day on A; hence

$$3 : 73 :: 1 : \frac{73}{3} \text{ first conjunction of A and B.}$$

$$3 : 2 \times 73 :: 1 : \frac{146}{3} \text{ second conjunction of A and B.}$$

$$3 : 3 \times 73 :: 1 : 73 \text{ third conjunction of A and B:}$$

this answer, being an integer, will be the number of days and nights.

(22)

$$\begin{array}{l} \text{As } \begin{array}{ccc} \text{£.} & \text{£.} & \text{£.} \\ 83 : 100 :: 63 \end{array} : \text{cost price;} \\ \text{and } 100 : 120 :: \text{cost price} : \text{trade price;} \\ \therefore 83 : 120 :: 63 : \text{trade price.} \end{array}$$

$$\therefore \text{trade price} = \frac{63 \times 120}{83} = \begin{array}{ccc} \text{£.} & \text{s.} & \text{d.} \\ 91 & 1 & 8\frac{1}{3} \end{array}$$

$$\begin{array}{r} 63 \quad 0 \quad 0 \\ \text{Ans. } \underline{28 \quad 1 \quad 8\frac{1}{3}} \end{array}$$

* If interest had been reckoned on the 117*l.* instead of discount, the selling price would have been a little more than 14*s.* 1*d.* per lb.

$$\begin{array}{r}
 (23) \\
 2s. \ 3d. \times 112 = 12l. \ 12s. = 12 \text{ guineas; then} \\
 \begin{array}{cccc}
 \text{gui.} & \text{gui.} & \text{£.} & \text{£.} \\
 12 & : 9 & :: 200 & : 150 \\
 & & & \underline{100} \\
 & & & 50 = \text{gain per cent.}
 \end{array}
 \end{array}$$

(24)
 The muslin cost 500*l.*, $\therefore \frac{1}{4}$ of it cost 125*l.*; now the buyer lost 50*l.* by this part of it; he therefore sold this part for 116*l.*; hence

As 5*s.* : 116*l.* :: 1 yd. : 466*l.* yds., the quantity damaged.

And 466*l.* $\times 2 = 933\frac{1}{2}$ yards, the quantity undamaged, and which now remains to be sold; and since 50*l.* is to be gained, we have 550 — 116*l.*, or 433*l.*, the sum for which the undamaged part must be sold.

Whence, 933*l.* yards : 5 qrs. :: 433*l.* : 11*s.* 7*d.* $\frac{1}{2}$ Ans.

(25)
 Since the hare runs 10 miles an hour, she will run 195*l.* yards in 40"; therefore when the chase begins, the hare has 195*l.* + 40, or 235*l.* yards' start of the dog. Now the dog runs 18 miles an hour, and gains 8 miles an hour upon the hare; we have therefore

As 8 miles : 235*l.* yards :: 1 hour : 60" $\frac{1}{2}$, the duration of the chase.

Also, 1 hour : 60" $\frac{1}{2}$:: 18 miles : 530 yards, the distance run by the dog.

(26)
 The first traveller starts 8 hours before the second, and goes at the rate of 3 miles an hour, he will therefore have advanced 24 miles before the second traveller begins his journey, at which time their distance will be 130 — 24, or 106 miles. They now approach each other at the rate of 7 miles per hour, and will therefore meet in $\frac{106}{7} = 15\frac{1}{7}$ hours. Whence $24 + 15\frac{1}{7} \times 3 = 69\frac{3}{7}$ miles = the distance required from Exeter.

(27)
 Since one of the cocks can fill the reservoir in 50 minutes, therefore in one minute this cock throws in $\frac{1}{50}$ th part necessary to fill the reservoir, and in one minute the

other cock throws in $\frac{1}{40}$ th part necessary to fill the reservoir. Now the discharging pipe can empty a $\frac{1}{5}$ th part of the reservoir; hence $\frac{1}{50} + \frac{1}{40} - \frac{1}{5} = \frac{1}{100}$ th part is really retained per minute; and since 200 of these parts will fill the reservoir, the time elapsed before the cistern is full will be 200 minutes, or 3 hours 20 minutes, the answer required.

(28)

Suppose the man had 7 sheep; then $7 + 7 + 3\frac{1}{2} + 7\frac{1}{2} = 25$.

And $25 : 20 :: 7 : 5$, omitting the fraction.

Note. In many cases of this sort the answer can only be found by taking the nearest integer, as in Ex. 7, in Single Position.

(29)

The four shares are as the numbers $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$; or 40, 30, 24, 20; the sum of which = 114.

$$\text{Hence } 114 : 40 :: \begin{array}{l} 40 \\ 30 \\ 24 \\ 20 \end{array} : \begin{array}{l} \overset{s.}{14} \quad \overset{d.}{0\frac{1}{8}} \text{ A's share.} \\ 10 \quad 6\frac{1}{8} \text{ B's share.} \\ 8 \quad 5\frac{3}{8} \text{ C's share.} \\ 7 \quad 0\frac{5}{8} \text{ D's share.} \end{array}$$

(30)

$$\begin{aligned} \overset{ft.}{60\frac{1}{2}} \times \overset{ft.}{33\frac{1}{2}} &= 60.5 \times 33.5 = 2026.75 = \text{area of floor.} \\ 15 \times 1\frac{1}{4} &= 15 \times 1.25 = 18.75 = \text{area of 1 plank.} \\ \therefore 2026.75 \div 18.75 &= 108\frac{7}{8} \text{ planks.} \end{aligned}$$

(31)

In this question we have only to divide 100*l.* into two parts which shall be to each other as 7 : 1, or so that the whole be to 1 part as 8 : 1;

$$\therefore 8 : 1 :: 100\text{.l.} : 12\text{.l. } 10\text{s.} = \frac{1}{8} \text{ of } 100\text{.l.}$$

Hence $7 \times 12\text{.l. } 10\text{s.}$, or 87*l. } 10\text{s.}*, is the principal required.

(32)

Since in one case the general has 284 soldiers too many, and in the other case too few by 25, therefore the difference of the squares must = $284 + 25 = 309$.

Take away the corner man of the rank and file; then $308 \div 2 = 154$ men in side of first square, and

$$154^2 + 248 = 24000 = \text{number of men.}$$

(33)

Now 5 is $\frac{1}{3}$ of 15, or $\frac{1}{3}$ of $2 \times 7\frac{1}{2}$, or $\frac{1}{3}$ of $7\frac{1}{2}$; hence $7\frac{1}{2}$ is that number, $\frac{1}{3}$ of which = 5. •

(34)

	£.	s.	d.		£.	s.	d.
	3179	11	8		2293	11	1 $\frac{1}{2}$
Add	25	0	0 for $\frac{1}{2}$ year.		100	0	0
13)	3204	11	8	4)	2393	11	1 $\frac{1}{2}$
	246	10	1 $\frac{1}{2}$		598	7	9 $\frac{1}{2}$
	2958	1	6 $\frac{1}{2}$		1795	3	4 $\frac{1}{2}$
	100	0	0		100	0	0
4)	3058	1	6 $\frac{1}{2}$	4)	1895	3	4 $\frac{1}{2}$
	764	10	4 $\frac{1}{2}$		473	15	10
	2293	11	1 $\frac{1}{2}$		1421	7	6 $\frac{1}{2}$

(35)

Since the person has $\frac{1}{3}$ his income + 15*l.* left, after he has spent $\frac{1}{3}$ of it + 10*l.*, therefore he has $\frac{1}{3}$ his income + 25*l.* left after spending $\frac{1}{3}$ of it; i. e., the difference between $\frac{1}{3}$ his income and $\frac{1}{3}$ of it = 25*l.* or $\frac{1}{3}$ of his income = 25*l.*; therefore his income = 150*l.*

(36)

The sharers are as the numbers $1\frac{1}{2}$, 1, 1, 1, 1, and $\frac{1}{2}$; or as 9, 6, 6, 6, 6, 2; the sum of which = 35.

Hence, as
 $35 : 212\text{ }l. \text{ } 14s. \text{ } 7d. :: \begin{cases} 9 : 54 \text{ } 14 & 0\frac{1}{2} \text{ the captain's share.} \\ 6 : 36 \text{ } 9 & 4\frac{1}{2} \text{ each man's share.} \\ 2 : 12 \text{ } 8 & 1\frac{1}{2} \text{ the boy's share.} \end{cases}$

(37)

The 1st cock discharges 60 gallons per hour.
 2nd cock discharges 30 gallons per hour.
 3rd cock discharges 20 gallons per hour.
 \therefore the three when all open discharge 110 gallons per hour;
 \therefore as 110 gall. : 60 gall. :: 60 min. : 32 $\frac{2}{3}$ min. *Ans.*

(38)

Now $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$; and $1 - \frac{1}{3} = \frac{2}{3}$ = the part occupied by cherry-trees; hence $50 \times 12 = 600$ = whole number of trees.

(39)

As $\frac{1}{3}$ of $\frac{1}{2} : 1$ (the whole mine) :: 1710*l.* : 3800*l.*

(40)

$5 \times 2\frac{1}{4}d. = 11\frac{1}{4}d. =$ daily cost of beef for 4 persons.

$\therefore \frac{12l. 12s.}{11\frac{1}{4}d.} \text{ or } \frac{12096}{45} = 268\frac{2}{3}$ messes of 4 men each;

$\therefore 1075\frac{1}{3} =$ number of men in the company.

Now $1\frac{1}{4}d. =$ cost of bread for 4 men per day.

$\therefore \frac{3}{4}d. =$ cost of bread for 1 man per day.

$\frac{3}{4}d. =$ cost of bread for 1 man per week.

$\therefore 1075\frac{1}{3} \times \frac{3}{4}d. = \frac{672 \times 21}{5}d. = 2822\frac{2}{3}d. =$ weekly cost of bread for the ship's company $= 11l. 15s. 2\frac{2}{3}d. \frac{2}{3} \text{ Ans.}$

(41)

By compound proportion,

$\frac{1\frac{1}{4}d.}{\frac{1}{4}} : \left\{ \begin{array}{l} 48d. \\ 1 \end{array} \right\} :: 6s. 7\frac{1}{4}d. \text{ or } 318 \text{ farthings; hence}$
 $\frac{318 \times 48 \times 1}{\frac{1}{4} \times \frac{1}{4}} = \frac{318 \times 4}{1\frac{1}{4}} = 318 \times 40 = 12720f. = 3180d. =$
 $265s. = 13l. 5s.$

(42)

Since A will do $\frac{1}{10}$ of the work in a day, and B will do $\frac{1}{15}$, \therefore A and B together can perform $\frac{1}{10} + \frac{1}{15}$, or $\frac{3}{10}$ of the work in a day.

Hence $\frac{3}{10} : 1 \text{ work} :: 1 \text{ day} : \frac{10}{3} \text{ days, or } 5\frac{2}{3} \text{ days. Ans.}$

(43)

Here B and C do $\frac{1}{15}$ of the work in one day; also A, B, and C, can perform $\frac{1}{10}$ of the work in a day, $\therefore \frac{1}{10} - \frac{1}{15}$ or $\frac{1}{30}$ = the part which A can perform daily; and $\frac{1}{30}$ or $28\frac{2}{3}$ days = the time in which alone A would do it.

(44)

In this question A can perform $\frac{1}{10}$ of the work in a day; A and B can jointly perform $\frac{1}{7}$; $\therefore \frac{1}{7} - \frac{1}{10}$ or $\frac{3}{70}$ is the part done by B in a day: $\therefore \frac{70}{3}$ or $23\frac{1}{3}$ days = the time B would require to perform the work.

(45)

Here, A, B, and C can jointly perform $\frac{1}{8}$ of the work in a day.

C can alone perform $\frac{1}{24}$ of the work in a day;

\therefore A and B can perform $\frac{1}{24}$ daily.

But A performs $\frac{1}{24}$ daily; therefore

B can perform $\frac{1}{24} - \frac{1}{24}$ or $\frac{1}{48}$ daily; and the time required for B to perform the work = $\frac{48}{1}$ days = 48 days.

(46)

Here the part done by A in a week = $\frac{1}{4}$.

the part done by B in a week = $\frac{1}{8}$.

the part done by C in a week = $\frac{1}{8}$.

$\therefore \frac{1}{4} + \frac{1}{8} + \frac{1}{8}$ or $\frac{1}{2}$ = the joint performance of all 3 in one week, and $\therefore \frac{1}{2}$ week = time consumed = 6 days, $5\frac{1}{2}$ hours.

(47)

120 oranges at 2 for a penny cost 60d.

120 oranges at 3 for a penny cost 40d.

total cost $\overline{100}$ pence.

Now since they are sold at 5 for 2d.; therefore

5 : 240 (number of oranges) :: 2d. : 96d. = the sum for which they were sold; which sum is less by 4d. than the cost price.

(48)

The quantity of water flowing per minute is $\frac{1}{4}$ gallons; and the quantity running out is $\frac{1}{8}$ gallons; and $\therefore \frac{1}{4} - \frac{1}{8}$ or $\frac{1}{8}$ gallons is the quantity accumulating every minute while both cocks are open; whence $\frac{1}{8} \times 180 = 47\frac{3}{4}$ gallons = the quantity in the tub when the discharging cock was stopped; hence $147 - 47\frac{3}{4}$ or $99\frac{1}{4}$ gallons = the quantity still wanting to fill the tub.

Also 14 gall. : $99\frac{1}{4}$ gall. :: 9 min. : 1 hr. 3 min. $48\frac{1}{4}$ sec; hence tub will be full at 3 min. 48 sec. past 6 o'clock.

(49)

Now A's share + B's share + C's share = 1500l.;

Or A's + (A's + 72) + (A's + 184) = 1500;

\therefore three times A's share + 256 = 1500l.;

Or three times A's share = 1244l.

\therefore A's share = $414\frac{2}{3}$ l.; hence B's share = $486\frac{1}{3}$ l., and C's share = $598\frac{1}{3}$ l.

(50)

Since A had 8 per cent. and B only 5, \therefore A had 3 per cent. allowed for his trouble:

$\therefore 8 + 5 : 3 :: 154 : \frac{154 \times 3}{8} = 35\frac{1}{2}$ 10s. $9\frac{1}{2}$ d. Ans.

(51)

Since C's share falls into the general fund, we must divide 100,000l. between A and B, in the proportion $\frac{1}{4} : \frac{1}{8}$ or 4 : 3.

Now 7 : 4 :: 100000l. : $57142\frac{2}{7}$ l. = A's share.

7 : 3 :: 100000l. : $42857\frac{1}{7}$ l. = B's share.

(52)

The stationer's gain being $\frac{2}{3}$ of 11s. the cost must have been $\frac{2}{3}$ of 11s. or $7\frac{1}{3}$ s.; now these quills he sold at 13s. 6d. or $13\frac{1}{2}$ or $13\frac{1}{2}$ s. per thousand; and therefore his gain in the latter case = $13\frac{1}{2} - 7\frac{1}{3} = 6\frac{1}{6}$ s. per thousand.

Hence as $6\frac{1}{6}$: $7\frac{1}{3}$:: 100l. : 96l. 7s. 3 $\frac{1}{4}$ d. *Ans.*

(53)

The least common multiple of the nine digits is readily found in the following manner.

$$2)1, 2, 3, 4, 5, 6, 7, 8, 9.$$

$$2)1, 1, 3, 2, 5, 3, 7, 4, 9.$$

$$3)1, 1, 3, 1, 5, 3, 7, 2, 9.$$

$$1, 1, 1, 1, 5, 1, 7, 2, 3.$$

$$\therefore 2 \times 2 \times 3 \times 5 \times 7 \times 2 \times 3 = 60 \times 42 = 2520. \text{ Ans.}$$

(54)

Since each calf which is 2 years old will produce a female calf at the end of each succeeding year, and that each of these last will produce a female calf at the expiration of 2 years, therefore the successive numbers of cattle in the farmer's stock will form a series of numbers found by adding the preceding term but one to the last term, and continued for 20 terms. Thus 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, 41, 60, 88, 129, 189, 277, 406, 595, 872, 1278; the last term of which series will be equal to the number of cattle in the farmer's stock at the end of the 20th year.

(55)

$$120 \text{ oz.} \times 48\text{s.} = 5760$$

$$120 \text{ oz.} \times 54\text{s.} = 6480$$

$$120 \text{ oz.} \times 60\text{s.} = 7200$$

$$\begin{array}{r} 360 \qquad 360 \overline{)19440} \end{array} (4\text{s. } 6\text{d. per ounce.}$$

(56)

	£.	s.	d.
The jewels cost in ready money	220	0	0
Interest for 8 mon. at 5 per cent. =	7	6	8
total cost =	227	6	8
As 102l. 10s. : 440l. :: 100l. : value of bill =	429	5	4 $\frac{1}{2}$
gain =	201	18	8 $\frac{1}{2}$

APPENDIX.

CASE I., p. 196.

To change old wine gallons into imperial gallons.

(2)

$$\begin{array}{r}
 \text{galls.} \quad \text{qts.} \\
 6) 106 \quad 3 \text{ old measure.} \\
 \underline{17} \quad 3\frac{1}{2} \\
 88 \quad 3\frac{1}{2} \text{ imp. measure.}
 \end{array}$$

CASE II., p. 196.

To change old ale or beer gallons into imperial gallons.

(2)

$$\begin{array}{r}
 \text{galls.} \quad \text{qts.} \quad \text{pts.} \\
 59) 376 \quad 2 \quad 0 \text{ old meas.} \\
 \underline{6} \quad 1 \quad 1\frac{2}{3} \\
 382 \quad 3 \quad 1\frac{2}{3} \text{ imp. meas.}
 \end{array}$$

CASE III., p. 196.

To change old corn gallons or bushels into imperial gallons.

(2)

$$\begin{array}{r}
 \text{bush.} \quad \text{pecks.} \\
 33) 129 \quad 2 \text{ old gallons.} \\
 \underline{3} \quad 3\frac{2}{3} \\
 125 \quad 2\frac{1}{3} \text{ imp. gallons.}
 \end{array}$$

CASE I., p. 197.

To change imperial gallons into old wine gallons.

(2)

$$\begin{array}{r}
 \text{galls.} \quad \text{qts.} \quad \text{pts.} \\
 5) 179 \quad 2 \quad 0 \text{ imp. meas.} \\
 \underline{35} \quad 3 \quad 1\frac{1}{2} \\
 215 \quad 1 \quad 1\frac{1}{2} \text{ old meas.}
 \end{array}$$

CASE II., p. 198.

To change imperial gallons into old ale or beer gallons.

(2)

$$\begin{array}{r}
 \text{galls.} \quad \text{qts.} \\
 60) 207 \quad 3 \text{ imp. meas.} \\
 \underline{3} \quad 1\frac{1}{2} \\
 204 \quad 1\frac{3}{4} \text{ old galls.}
 \end{array}$$

CASE III., p. 198.

To change imperial bushels into old corn bushels.

(2)

$$\begin{array}{r}
 \text{bush.} \quad \text{pkts.} \quad \text{galls.} \\
 32) 279 \quad 3 \quad 1 \text{ imp. meas.} \\
 \underline{8} \quad 2 \quad 1\frac{2}{3} \\
 288 \quad 2 \quad 0\frac{2}{3} \text{ old meas.}
 \end{array}$$

EXAMPLES TO TABLE I. IN THE APPENDIX.

CASE I.

(2)

Opposite 44 in the table we find 8·55715 ;
 then $8\cdot55715 \times 500 = 4278\cdot575\text{ l.}$
 $= 4278\text{ l. } 11\text{ s. } 6\text{ d.}$

CASE II.

(2)

$\frac{1\cdot5\cdot0\cdot0}{7\cdot5} = 7\cdot5$: search the table for a number nearly =
 7·5, and you will find 7·391989, opposite to which is 41,
 which is nearly equal to the number of years required.

(3)

In this example the amount is to become equal to three
 times the principal, therefore the former divided by the
 latter = 3. Now the number in the table nearest to 3 is
 3·071523, opposite to which stands 23, the number of years
 required.

(4)

$\frac{1\cdot0\cdot0\cdot0}{11\cdot627} = 11\cdot627$; the number in the table nearest to
 11·627 is 11·467, opposite to which is 50 years. Hence
 the time required will be about 50½ years.

(5)

$\frac{100}{52\cdot5} = 1\cdot904$; the number in the table nearest to this, is
 1·9799, which points out 14 years; hence the time sought
 is rather less than 14 years.

EXAMPLES SHOWING THE USE OF TABLE II.

CASE I.

(2)

Opposite 21 in the column of years you find 35·7192,
 which is the amount of an annuity of 1l. for 21 years :
 hence the amount of an annuity of 50l. for 21 years
 $= 35\cdot7192 \times 50 = 1785\cdot96\text{ l.} = 1785\text{ l. } 19\text{ s. } 2\text{ d.}$

(3)

Opposite 23 in the column of years you find 41·4305,
 which is the amount of an annuity of 1l. for 23 years ;
 hence $41\cdot4305 \times 100 = 4143\cdot05 = 4143\text{ l. } 1\text{ s.}$ the amount
 required.

CASE II.

(2)

$1\frac{2}{3}\% = 40$; now in the column of amounts you find 41·4305, which is the amount in 23 years; hence the time required will be nearly $22\frac{1}{2}$, which may be found by a simple proportion, exact enough for all common purposes.

(3)

$\frac{566698\cdot7}{1212} = 467\cdot57$; look for a number nearest to this

in the column of amounts, and you will find 456·798 which being somewhat less than 467·57, shows that the time sought is more than 65 years.

EXAMPLES SHOWING THE USE OF TABLE III.

(2)

The present value of 1*l.* per annum for 14 years = 9·898641; hence $9\cdot898641 \times 150 = 1484*l.* 15*s.* 11*d.*$

(3)

The present value of 1*l.* per annum for 84 years = 19·668; hence $19\cdot668 \times 45 = 885*l.* 1*s.* 2\frac{1}{2}$ *d.*

(4)

The present value of 1*l.* per annum for 12 years = 8·863252; hence $8\cdot863252 \times 750 = 6647*l.* 8*s.* 9\frac{1}{2}$ *d.*

EXPECTATION OF LIFE.

EXAMPLES SHOWING THE USE OF TABLE III.

(2)

Opposite 76 in the column of ages you will find 4·511, which is the value of an annuity of 1*l.* during a life aged 76 years: hence $75 \times 4\cdot511 = 338*l.* 6*s.* 6*d.*$ the sum required.

(3)

The yearly income derived from 1250*l.* in the 3 per cents. = 37*l.* 10*s.* and the value of an annuity of 1*l.* during the life of a person aged 53 = 9·748; hence the value required = $37\cdot5 \times 9\cdot748 = 365*l.* 11*s.*$ *Ans.*

(4)

The actual value of 3000*l.* in the 5 per cents at 98 is 2940*l.*; also the value of an annuity of 1*l.* during a life aged 57 years is 8·999; $\therefore \frac{2940}{8\cdot999} = 326*l.* 14*s.*$ the yearly annuity required.

(5)

The value of an annuity of 100*l.* per annum during the life of a person aged 32 = $100 \times 12\cdot854 = 1285\cdot4*l.*$; and the value of 100*l.* per annum for 25 years certain = $14\cdot093945 \times 100$ (see Table III. p. 204) = 1409·39*l.* and hence $1409\cdot3 - 1285\cdot4 = 123\cdot9*l.* = 123*l.* 18*s.*$ the answer required.

EXAMPLE SHOWING THE USE OF TABLE IV.

(2)

Value of a single life aged 15 years (p. 208)	= 14·588
Value of a single life aged 10 years (ditto)	= 15·139
Sum of the values of these single lives . . .	= 29·727
By Table IV. the value of their joint continuance	= 12·302
Value of the longer of the two lives	= 17·425

Hence, since the value of the two lives is worth 17·425 years' purchase, the value required = 1742*l.* 10*s.*

EXAMPLES ON INSURANCE.

(2)

The value of a life aged 45 years = 11·105 by Table III.; and the perpetuity = 20 years' purchase:

$$\therefore 20 - 11\cdot105 = 8\cdot895$$

and $\therefore \frac{8\cdot895 \times 1000}{21} = \frac{8895}{21} = 423*l.* 11*s.* 5*d.*$ = the sum to be paid down in order to assure 1000*l.* to his children at his decease: also $\frac{423*l.* 11*s.* 5*d.*}{11\cdot105} = 38*l.* 2*s.* 10*d.* = the annual cost;$

supposing also that the first annual payment is not made at the time of effecting the policy.

(3)

The value of a life aged 25 years is 13·567 ;

Also $20 - 13·567 = 6·433$;

$\therefore \frac{6·433 \times 1000}{21} = \frac{6433}{21} = 306\text{l. } 6\text{s. } 8\text{d.}$ the cost of the assurance if only one payment be made.

Also $\frac{306\text{l. } 6\text{s. } 6\text{d.}}{14·567} = 21\text{l.}$ nearly, by annual payments.

(4)

The value of a life aged 60 years is 8·392 .

$\therefore 20 - 8·392 = 11·608$,

Hence $\frac{11·608 \times 1000}{21} = \frac{11608}{21} = 552\text{l. } 15\text{s. } 2\frac{1}{2}\text{d.}$ = the sum to be paid immediately in one payment.

Also $\frac{552\text{l. } 15\text{s. } 2\frac{1}{2}\text{d.}}{9·392} = 59\text{l.}$ very nearly, the annual cost of assuring 1000\text{l.} to be paid at the death of an individual aged 60 years ; the first yearly payment being paid immediately.

EXAMPLES IN THE VALUES OF LEASES.

(2)

200\text{l.} - 49\text{l. } 12\text{s.} = 150\text{l. } 8\text{s.} = \text{annual value of the house to the purchaser. Now in the table opposite to 14 and in the column of 6 per cent. you will find } 9·2949 ; \text{ hence } 150·4 \times 9·2949 = 1397\text{l. } 19\text{s. } 0\frac{1}{2}\text{d.} the value required.

To find the Annual Rent corresponding to a given sum paid for a Lease.

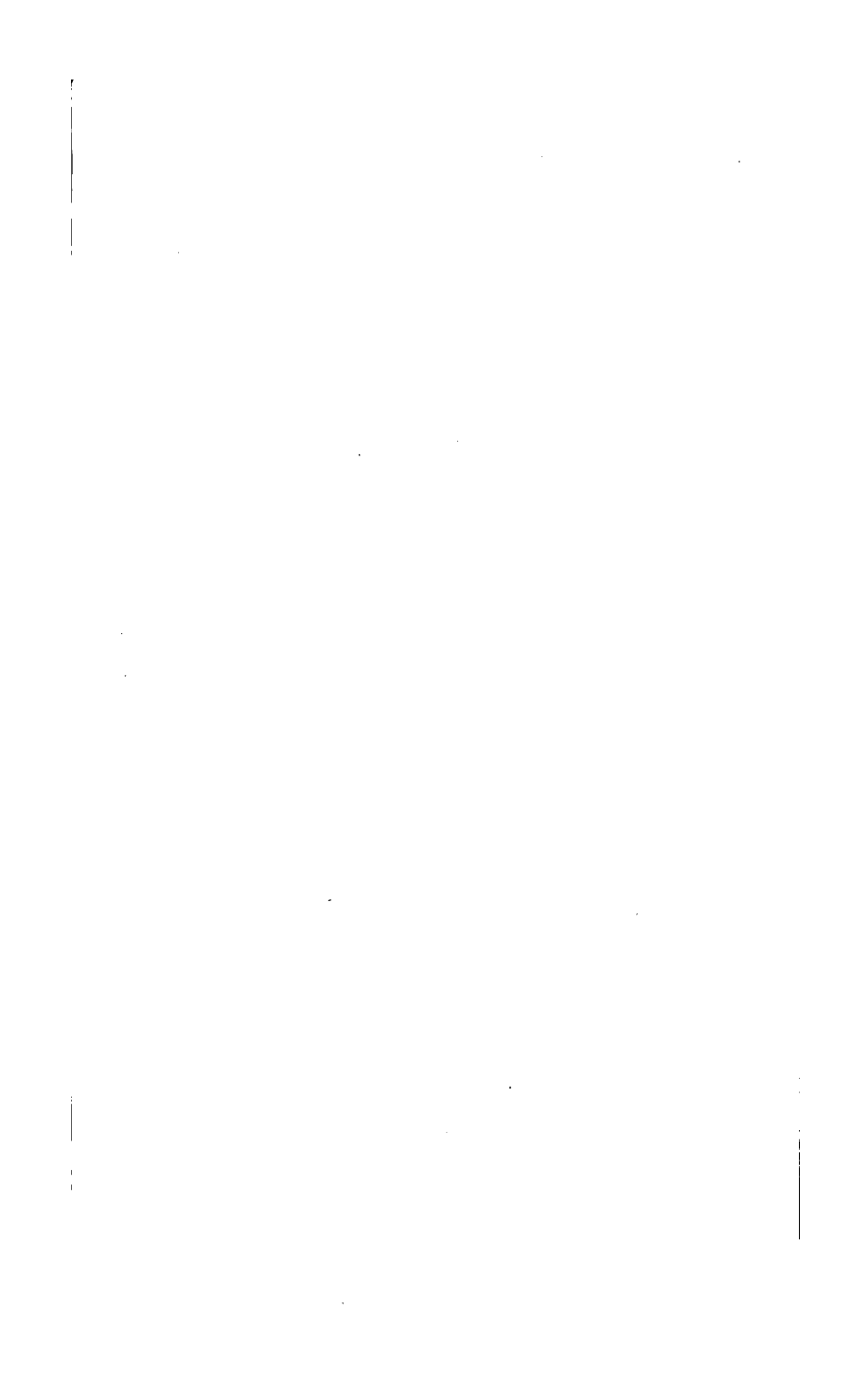
(2)

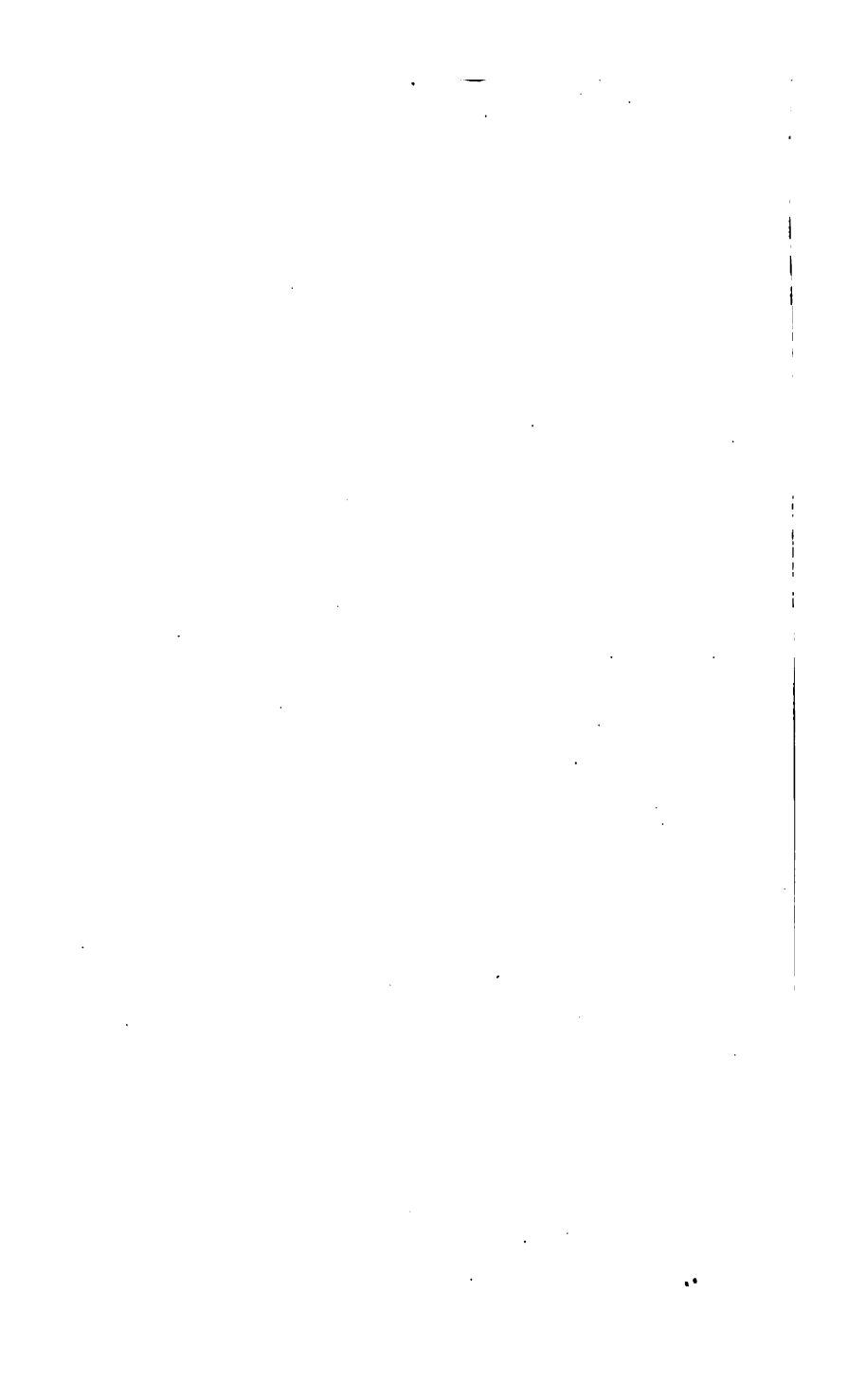
In this example the term is 81 years, and the yearly rent = 840\text{l.} ; $\therefore \frac{840}{14·226} = 59\text{l. } 0\text{s. } 9\text{d.}$, nearly ; to this I must add 5\text{l. } 5\text{s.} the ground rent, and the house will then cost the purchaser 64\text{l. } 5\text{s. } 9\text{d.} annually.

THE END.

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